Generalização da fórmula da corrente para multi-terminais

$$I_\alpha = -\frac{e}{h}\sum_\sigma \int dE \sum_\beta tr\left[\delta_{\beta\alpha} - S^\dagger_{\alpha\beta} S_{\beta\alpha}\right] f_\beta(E)$$

na forma

$$I_\alpha = -\sum_{\beta,\beta\neq\alpha} g_{\alpha\beta}(V_\alpha - V_\beta)$$

precisamos

$$f_\beta(E) = f_\alpha(E) + \left.\frac{\partial f}{\partial \mu}\right|_{\mu_\alpha} (\mu_\beta - \mu_\alpha) + \mathcal{O}(\Delta\mu^2) \qquad \mu_\beta = -eV_\beta \quad \frac{\partial f}{\partial \mu} = -\frac{\partial f}{\partial \epsilon}$$

$$= f_\alpha(\epsilon) + e\left(-\frac{\partial f}{\partial \epsilon}\right)\Big|_{\mu=\mu_\alpha} (V_\alpha - V_\beta)$$

daí

$$g_{\alpha\beta} = \underbrace{2_s\frac{e^2}{h}}_{G_0} \int dE \; tr\left[\delta_{\alpha\beta} - S^\dagger_{\alpha\beta} S_{\beta\alpha}\right]\left(-\frac{\partial f_\alpha}{\partial \epsilon}\right)$$

$$tr\left[\delta_{\alpha\beta} - S^\dagger_{\alpha\beta} S_{\alpha\beta}\right] = \begin{cases} N_\alpha - R_\alpha(E) = N_\alpha - tr(r^\dagger_\alpha r_\alpha) & \text{se } \alpha = \beta \\ -T_{\alpha\beta}(E) = -tr(t^\dagger_{\alpha\beta} t_{\alpha\beta}) & \text{se } \alpha \neq \beta \end{cases}$$

Definimos a probabilidade de transmissão e reflexão como

$$\mathcal{T}_{\alpha\beta} = \int dE \; T_{\alpha\beta}(E)\left(-\frac{\partial f_\alpha}{\partial E}\right) \quad e \quad \mathcal{R}_\alpha = \int dE \; R_\alpha(E)\left(-\frac{\partial f_\alpha}{\partial E}\right)$$

Daí

$$\begin{pmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{pmatrix} = 2_s\frac{e^2}{h} \begin{pmatrix} N_1 - \mathcal{R}_1 & -\mathcal{T}_{12} & \cdots & -\mathcal{T}_{1n} \\ -\mathcal{T}_{21} & N_2 - \mathcal{R}_2 & \cdots & -\mathcal{T}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ -\mathcal{T}_{n1} & \mathcal{T}_{n2} & \cdots & N_n - \mathcal{R}_n \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{pmatrix}$$

$$N_\alpha - R_\alpha = \sum_{\beta, \beta\neq\alpha} T_{\alpha\beta}$$

unitariedade de S

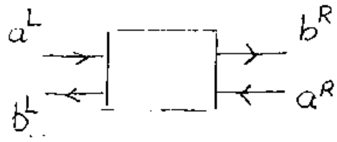
$$\text{e} \quad N_\alpha - R_\alpha = \sum_{\alpha, \alpha\neq\beta} T_{\alpha\beta}$$

Contato voltimétrico (corrente é nula)

$$I_\alpha = 0 \qquad \sum_{\beta, \beta\neq\alpha} G_{\alpha\beta}\,(V_\alpha - V_\beta) = 0$$

$$V_\alpha = \frac{\sum_{\beta(\neq\alpha)} T_{\alpha\beta} V_\beta}{\sum_{\gamma(\neq\alpha)} T_{\alpha\gamma}}$$

Aviso: Matriz T de espalhamento é outra coisa!



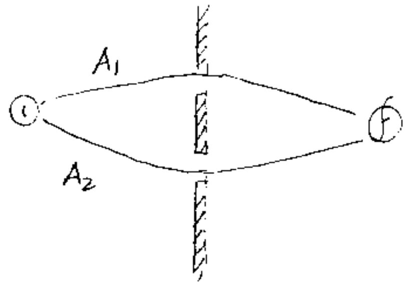
$$\begin{pmatrix} b^R \\ a^R \end{pmatrix} = T \begin{pmatrix} a^L \\ b^L \end{pmatrix}$$

como vimos $$\begin{pmatrix} b^L \\ b^R \end{pmatrix} = S \begin{pmatrix} a^L \\ a^R \end{pmatrix} \qquad S = \begin{pmatrix} r^L & t^{LR} \\ t^{RL} & r^R \end{pmatrix}$$

então $$T = \begin{pmatrix} (t^{LR}t^{RL} - r^L r^R)/t^{LR} & r^R/t^{LR} \\ -r^L/t^{LR} & 1/t^{LR} \end{pmatrix}$$

Interferência Quântica

Idéia geral



caminho 1 ... amplitude de probabilidade A_1

caminho 2 ... amplitude de probabilidade A_2

clássico $\quad P_{cl} = P_1 + P_2 \qquad P_\alpha = |A_\alpha|^2$

quântico $\quad P_{qm} = |A_1 + A_2|^2 = |A_1|^2 + |A_2|^2 + A_1 A_2^* + A_1^* A_2$

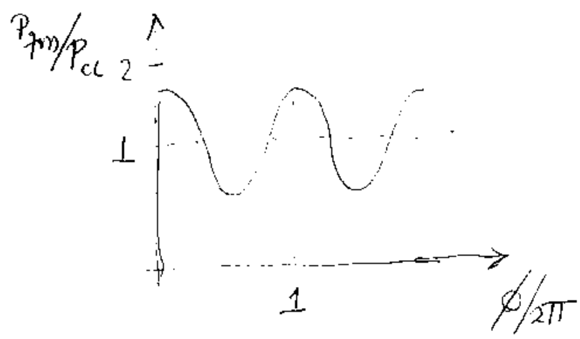
$$= P_{cl} + 2 Re(A_1 A_2^*)$$

Vamos supor que $\quad A_1 = \sqrt{P_1}\, e^{i\phi_1}$ e $A_2 = \sqrt{P_2}\, e^{i\phi_2}$

$$Re\, A_1 A_2^* = \sqrt{P_1 P_2}\, Re\, e^{i(\phi_1 - \phi_2)} = \sqrt{P_1 P_2} \cos\phi$$

deslocamento de fase

$$P_{qm} = 2\sqrt{P_1 P_2} \cos\phi$$



$$0 \leqslant \frac{\sqrt{P_1 P_2}}{P_{cl}} \leqslant 1$$

Deslocamento de fase

Na ausência de espalhamento, semiclassicamente

$$\psi(x) = e^{i\phi(x)} \qquad \frac{d\phi}{dx} = k(x) \equiv \sqrt{2m(E-V(x))}/\hbar$$

vetor de onda "local"

$$\psi(x) = e^{i/\hbar \int_{x_i}^{x_f} dx\, p(x)} \qquad \phi_{din} = \frac{1}{\hbar}\int_{\vec{r}_i}^{\vec{r}_f} d\vec{r}\cdot\vec{p}$$

para $V(x) = cte$, $\phi(x) = k(x_f - x_i) = kL$

No caso de campo magnético é mais interessante: efeito Aharonov-Bohm

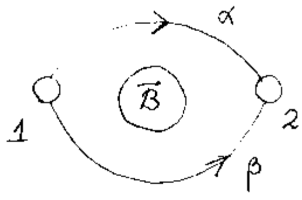
Fase acumulada:

$$\vec{p} \to \vec{p} + \frac{e}{c}\vec{A}$$

$$\phi_{mag} = \frac{e}{\hbar c}\int_{\vec{r}_1}^{\vec{r}_2} d\vec{x}\cdot\vec{A}$$

integral de linha

invariância de calibre $\vec{A} \to \vec{A} + \vec{\nabla}\alpha$ (não observável)



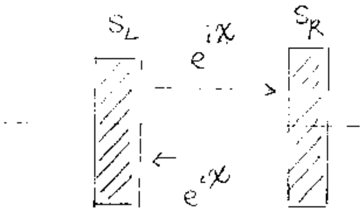
$$\phi_{mag}^{1\to 2,\alpha} = \frac{e}{\hbar c}\int_{1,\alpha}^{2} d\vec{x}\cdot\vec{A}$$

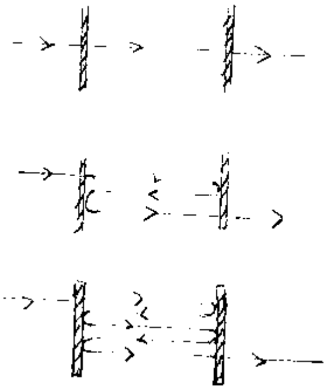
$$\phi_{max}^{1\to 2,\beta} = \frac{e}{\hbar c}\int_{1,\beta}^{2} d\vec{x}\cdot\vec{A}$$

$$\Delta\phi_{max} = \phi_{mag}^{\alpha} - \phi_{mag}^{\beta} = \frac{e}{\hbar c}\oint d\vec{x}\cdot\vec{A} = \frac{e}{\hbar c}\int d\vec{S}\cdot\vec{B} = 2\pi\frac{\phi}{\phi_0}$$

onde $\phi_0 = h/e$ (alguns autores 50% usam $\phi_0 = h/2e$)

Dupla junção





$$A_1 = t_L e^{i\chi} t_R$$

$$A_2 = t_L e^{i\chi}\left(r_R e^{i\chi} r_L' e^{i\chi}\right) t_R$$

$$A_3 = t_L e^{i\chi}\left(r_R e^{i\chi} r_L' e^{i\chi}\right)^2 t_R$$

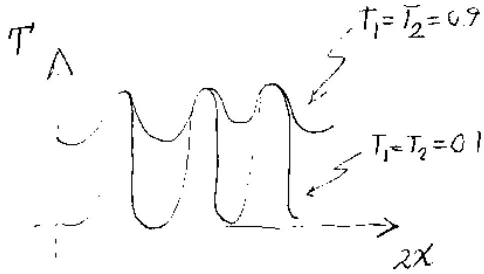
$$S_L = \begin{pmatrix} r_L & t_L' \\ t_L & r_L' \end{pmatrix}$$

$$S_R = \begin{pmatrix} r_R & t_R' \\ t_R & r_R' \end{pmatrix}$$

$$A_n = t_L t_R (r_L' r_R)^{n-1} e^{i(2n-1)\chi}$$

$$t = \sum_{n=1}^{\infty} A_n = t_L t_R e^{i\chi} \sum_{m=0}^{\infty}\left(r_L' r_R e^{i2\chi}\right)^m$$

$$t = \frac{t_R t_L e^{i\chi}}{1 - r_L' r_R e^{i2\chi}}$$



Transmissão

$$T = \left|\frac{t_R t_L e^{i\chi}}{1 - r_L' r_R e^{i2\chi}}\right|^2 = \frac{T_R T_L}{1 + R_L R_R - 2\sqrt{R_L R_L}\cos 2\chi}$$

$\hookrightarrow 2\chi \to 2\chi + \arg(r_L' r_R)$

pequeno "roubo"

$$T_{min} = \frac{T_R T_L}{(1+\sqrt{R_L R_R})^2} \leq T(\chi) \leq \frac{T_R T_L}{(1-\sqrt{R_L R_R})^2} = T_{max}$$

se $T_{L,R} \ll 1$ $\quad R_{L,R} = 1 - T_{L,R}$

$$T_{min} \approx T_R T_L \ll 1 \quad \text{e} \quad T_{max} = \frac{T_R T_L}{\left[1-\sqrt{(1-T_L)(1-T_R)}\right]^2} \approx \frac{T_R T_R}{\left[1-\sqrt{1-(T_L+T_R)+T_R T_L}\right]^2}$$

$$T_{max} \approx \frac{4 T_R T_L}{(T_L+T_R)^2}$$

Fabre - Perot