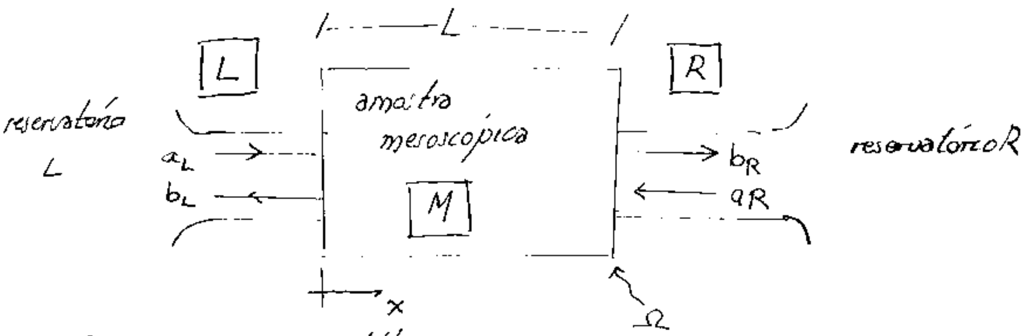
8. Espalhamento no formalismo de operadores

Voltemos ao problema



função de onda assintótica

$$\phi^{\pm}_{\alpha n E}(x,\vec{r}_\perp) = \frac{1}{\sqrt{2\pi\hbar v_n(E)}}\, e^{\pm i k_n(E) x}\, \chi_{n\alpha}(\vec{r}_\perp) \qquad (x,\vec{r}_\perp)\in\alpha \qquad \alpha = R, L$$

$$k_n = \sqrt{\frac{2m}{\hbar^2}(E-\varepsilon_n)}$$

função de onda total

$$\psi_E(x,\vec{r}_\perp) = \begin{cases} \sum_n a_{nL}\, \phi^{+}_{LnE}(x,\vec{r}_\perp) + \sum_n b_{nL}\, \phi^{-}_{LnE}(x,\vec{r}_\perp) & (x,\vec{r}_\perp)\in L \\ \psi_{ME}(x,\vec{r}_\perp) & (x,\vec{r}_\perp)\in M \\ \sum_n b_{nR}\, \phi^{+}_{RnE}(x,\vec{r}_\perp) + \sum_n a_{nR}\, \phi^{-}_{LnE}(x,\vec{r}_\perp) & (x,\vec{r}_\perp)\in R \end{cases}$$

Solução (casamento de função de onda) — $4\times N$ equações acopladas

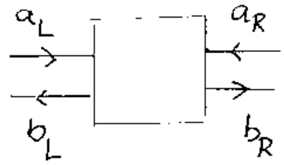
$$a_{nL} + b_{nL} = \sqrt{2\pi\hbar v_n(E)} \int_\Omega d\vec{r}_\perp\, \chi_n(\vec{r}_\perp)\, \psi_{ME}(0,\vec{r}_\perp)$$

$$a_{nR}\, e^{-ik_nL} + b_{nR}\, e^{ik_nL} = \sqrt{2\pi\hbar v_n(E)} \int_\Omega d\vec{r}_\perp\, \chi_n(\vec{r}_\perp)\, \psi_{ME}(L,\vec{r}_\perp)$$

$$a_{nL} - b_{nL} = \frac{1}{i\sqrt{2\pi\hbar v_n(E)}} \int_\Omega d\vec{r}_\perp\, \chi_n(\vec{r}_\perp)\, \left.\frac{\partial\psi_{ME}(x,\vec{r}_\perp)}{\partial x}\right|_{x=0}$$

$$b_{nR}\, e^{ik_nL} - a_{nR}\, e^{-ik_nL} = \frac{1}{i\sqrt{2\pi\hbar v_n(E)}} \int_\Omega d\vec{r}_\perp\, \chi_n(\vec{r}_\perp)\, \left.\frac{\partial\psi_{ME}(x,\vec{r}_\perp)}{\partial x}\right|_{x=L}$$

Queremos escrever o operador de campo que corresponde a partículas associadas aos estados de espalhamentos discutidos. Para isto vamos associar operadores de criação e aniquilação aos elétrons que incidem (a^+ e a) e emergem (b^+ e b) da região central, o condutor. Estes obedecem a



$$b_{\alpha l\sigma}(E) = \sum_{\alpha=L,R}\sum_{l'} S_{\alpha l,\beta l'}(E)\, a_{\beta l'\sigma}(E)$$

$$b^+_{\alpha l\sigma}(E) = \sum_{\alpha=L,R}\sum_{l'} S_{\beta l',\alpha l}(E)\, a^+_{\beta l'\sigma}(E)$$

Como são férmions

$$\left\{a^+_{\alpha l\sigma}(E), a_{\beta l'\sigma'}(E')\right\} = \delta_{\alpha\beta}\,\delta_{ll'}\,\delta_{\sigma\sigma'}\,\delta(E-E')$$

$$\left\{a^+_{\alpha l\sigma}(E), a^+_{\alpha' l'\sigma'}(E')\right\} = 0$$

Mostre que b e b^+ satisfazem a regra de anti-comutação.

Com estes elementos podemos escrever os operadores de campo $\hat{\psi}_\sigma(\vec{r},t)$ e $\hat{\psi}^+_\sigma(\vec{r},t)$ que aniquilam e criam elétrons com projeção de spin σ na posição $\vec{r}$ e tempo t.

Para o fio (guia de onda) à esquerda

$$\hat{\psi}_\sigma(\vec{r},t) = \int dE\, e^{-iEt/\hbar} \sum_n \left[a_{Ln\sigma}\, \phi^+_{LnE}(x,\vec{r}_\perp) + b_{Ln\sigma}\, \phi^-_{LnE}(x,\vec{r}_\perp) \right]$$

$$\hat{\psi}^+_\sigma(\vec{r},t) = \int dE\, e^{+iEt} \sum_n \left[a^+_{Ln\sigma}\, \phi^-_{LnE}(x,\vec{r}_\perp) + b^+_{Ln\sigma}\, \phi^+_{LnE}(x,\vec{r}_\perp) \right]$$

o operador corrente no fio L é

$$\hat{I}(x_L,t) = \frac{\hbar e}{2im}\sum_\sigma \int d\vec{r}_\perp \left[\hat{\varphi}^+_\sigma \frac{\partial}{\partial x_L}\hat{\varphi}_\sigma - \left(\frac{\partial}{\partial x_L}\hat{\varphi}^+_\sigma\right)\hat{\varphi}_\sigma \right]$$

$$\int d\vec{r}_\perp \hat{\varphi}^+_\sigma \frac{\partial}{\partial x_L}\hat{\varphi}_\sigma = \int dE \int dE' e^{-i(E-E')t/\hbar} \sum_{nn'} \int d\vec{r}_\perp \left(a^+_{Ln'\sigma}\phi^-_{Ln'E'} + b^+_{Ln'\sigma}\phi^+_{Ln'E'} \right)$$
$$\times \left(ik_n a_{Ln\sigma}\phi^+_{LnE} - ik_n b_{Ln\sigma}\phi^-_{LnE} \right)$$

$$\left\langle e^{i(E'-E)t/\hbar} \right\rangle_{\Delta t} \approx 2\pi\hbar\, \delta(E-E')$$

$$\left\langle \int d\vec{r}_\perp \hat{\varphi}^+_\sigma \frac{\partial}{\partial x_L}\varphi_\sigma \right\rangle_{\Delta t} = 2\pi\hbar \sum_n \int dE \frac{1}{2\pi\hbar v_n(E)} \cdot ik_n \left(a^+_{Ln\sigma} + b^+_{Ln\sigma}\right)\left(a_{Ln\sigma} - b_{Ln\sigma}\right)$$
$$= \frac{im}{\hbar}\sum_n \int dE \left(a^+_{Ln\sigma} + b^+_{Ln\sigma}\right)\left(a_{Ln\sigma} - b_{Ln\sigma}\right)$$

$$\langle \hat{I} \rangle_{\Delta t} = \frac{e}{2}\sum_\sigma\sum_n \int dE \Big[\left(a^+_{Ln\sigma}a_{Ln\sigma} - a^+_{Ln\sigma}b_{Ln\sigma} + b^+_{Ln\sigma}a_{Ln\sigma} - b^+_{Ln\sigma}b_{Ln\sigma} \right)$$
$$\left(a^+_{Ln\sigma}a_{Ln\sigma} - b^+_{Ln\sigma}a_{Ln\sigma} + a^+_{Ln\sigma}b_{Ln\sigma} - b^+_{Ln\sigma}b_{Ln\sigma} \right)$$

$$= e\sum_\sigma\sum_n \int dE \left[a^+_{Ln\sigma}(E)a_{Ln\sigma}(E) - b^+_{Ln\sigma}(E)b_{Ln\sigma}(E) \right]$$

$$= e\sum_{n\sigma}\sum_{\alpha\beta\ell\ell'} \int dE\; a^+_{\alpha\ell\sigma}(E)a_{\beta\ell'\sigma}(E)\left[\delta_{\alpha L}\delta_{\beta L}\delta_{n\ell}\delta_{n\ell'} - S^*_{\alpha\ell,Ln}(E)S_{Ln\beta n'}(E) \right]$$

Finalmente a média térmica $\langle a^+_{\alpha\ell\sigma}(E)a_{\beta\ell'\sigma}(E)\rangle = \delta_{\alpha\beta}\delta_{\ell\ell'}f_\alpha(E)$

$$\langle \hat{I}_L \rangle = e\sum_\sigma \int dE \underbrace{\sum_{\beta mn} \left[\delta_{\beta L}\delta_{mn} - |S_{Ln,\beta m}|^2 \right]}_{\sum_\beta tr\left[\delta_{\beta L} - S^+_{L\beta}S_{L\beta}\right]} f_\beta(E)$$

encontrar fator $1/h$ faltante!

9. Circuitos multi-terminais

generalização da fórmula de Landauer (resposta linear!)

$$\begin{pmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{pmatrix} = \begin{pmatrix} g_{11} & g_{12} & \cdots & g_{n1} \\ g_{21} & g_{22} & \cdots & g_{n2} \\ \vdots & & \ddots & \vdots \\ g_{n1} & g_{n2} & & g_{nn} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{pmatrix}$$

i) Conservação de carga: No regime estacionário $\sum_{i=1}^{n} I_i = 0$, lei de Kirchhoff

$$\sum_{i=1}^{n} g_{ij} = 0$$

ii) Corrente nula em equilíbrio. Para $V_i = V_0$, qualquer $i = 1, \ldots, n$ as correntes se anulam $I_i = 0$. Portanto

$$\sum_{i=1}^{n} g_{ji} = 0$$

Portanto $$I_i = \sum_j g_{ij} V_j = \sum_{\substack{j \\ j \neq i}} g_{ij} V_j + g_{ii} V_i$$

$$g_{ii} = -\sum_{\substack{j \\ j \neq i}} g_{ij}$$

$$I_i = -\sum_{\substack{j \\ j \neq i}} g_{ij} \underbrace{(V_i - V_j)}_{\text{depende apenas de diferenças de voltagem}}$$

$$G_{\alpha\beta} = \frac{e^2}{h} tr\left[\delta_{\alpha\beta} - S^{\dagger}_{\alpha\beta} S_{\alpha\beta}\right]\left(-\frac{\partial f_\beta}{\partial \epsilon}\right)$$

Para $\alpha \neq \beta$ $\quad tr[\delta_{\alpha\beta} - S^{\dagger}_{\alpha\beta} S_{\alpha\beta}] = tr[t^{\dagger}_{\alpha\beta} t_{\alpha\beta}] = T_{\alpha\beta}$

coeficente de transmissão

Para $\alpha = \beta$ $\quad tr[\delta_{\alpha\beta} - S^{\dagger}_{\alpha\beta} S_{\alpha\beta}] = tr[\mathbb{1} - r^{\dagger}_{\alpha\alpha} r_{\alpha\alpha}]$

$$= N_\alpha - tr[r^{\dagger}_{\alpha\alpha} r_{\alpha\alpha}]$$

$$= N_\alpha - R_\alpha$$

coeficente de reflexão

Pela unitariedade da matrix S, $\quad N_\alpha = R_\alpha + \sum_{\beta, \beta\neq\alpha} T_{\alpha\beta}$

o que está de acordo com $\sum_{\alpha=1}^{n} G_{\alpha\beta} = 0$

Portanto

$$\begin{pmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{pmatrix} = \frac{e^2}{h} \begin{pmatrix} N_1 - R_1 & -T_{12} & & \\ -T_{21} & N_2 - R_2 & & \\ & & \ddots & \\ & & & N_n - R_n \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{pmatrix}$$

onde $\quad T_{\alpha\beta} = g_s \int dE\, T_{\alpha\beta}(E)\, \frac{\partial f_\beta}{\partial \mu}$

Matriz T é outra coisa

$$\begin{pmatrix} b^R \\ a^R \end{pmatrix} = T \begin{pmatrix} a^L \\ b^L \end{pmatrix}, \quad \text{se } S = \begin{pmatrix} r^L & t^{LR} \\ t^{RL} & r^R \end{pmatrix} \therefore T = \begin{pmatrix} (t^{LR} t^{RL} - r^L r^R)/t^{LR} & r^R/t^{LR} \\ -r^L/t^{LR} & 1/t^{LR} \end{pmatrix}$$