



Oscillating magneto- and electrocaloric effects on bilayer graphenes



Z.Z. Alisultanov^{a,b,c,*}, M.S. Reis^d

^a Amirkhanov Institute of Physics Russian Academy of Sciences, Dagestan Science Centre, Makhachkala, Russia

^b Prokhorov General Physics Institute Russian Academy of Sciences, Moscow, Russia

^c Dagestan States University, Makhachkala, Russia

^d Instituto de Física, Universidade Federal Fluminense, Av. Gal. Milton Tavares de Souza s/n, 24210-346, Niterói, RJ, Brazil

ARTICLE INFO

Article history:

Received 20 October 2014

Received in revised form

6 December 2014

Accepted 4 January 2015

Communicated by C. Lacroix

Available online 17 January 2015

Keywords:

Bilayer graphene

Oscillating magnetocaloric effect

ABSTRACT

Recently, the oscillating magnetocaloric effect (OMCE) was proposed and theoretical verified as an intrinsic property of diamagnetic materials; and then it was further optimized considering a monolayer graphene. OMCE biased by an electric field was also deeply investigated, but Joule heating was a problem to be overcome. The present effort thus goes further and analysis of the OMCE of biased bilayer graphene under an electric and magnetic field is carried out. General expressions for the thermodynamic potentials are derived and then the entropy change and the adiabatic temperature change are investigated. We found that the results for bilayer graphenes are very similar to those for monolayer and, in addition, the Joule heating problem could be overcome, since the gate potential is applied between the two layers.

© 2015 Elsevier Ltd. All rights reserved.

1. Introduction

The magnetocaloric effect (MCE) is an interesting property in which magnetic materials, under a magnetic field change, are able to exchange heat with a thermal reservoir (considering an isothermal process), or even change its temperature (considering an adiabatic process). This effect is completely analogous to the compression–expansion thermal-mechanical cycle; and therefore the main purpose of the scientific community is to build a thermo-magnetic machine, to substitute, in a near future, those standard, non-economical and non-environmental friendly Freon-like refrigerators [1,2]. It is straightforward to see that $\Delta T = T_f - T_i$, obtained from the adiabatic condition $S(T_i, 0) = S(T_f, H)$, characterizes the adiabatic process; while $\Delta S = S(T, H) - S(T, 0)$ characterizes the magnetic entropy change, obtained from the isothermal process.

The magnetic entropy change is larger in magneto-ordered materials in the vicinity of phase transitions [1,3] and therefore the strongest MCE manifests in ferro- and antiferromagnetic materials [1–6]. Due to the above, a deeper evaluation of this effect in diamagnetic materials was only reported recently [7–17]; namely in 3D, 2D and quasi-1D non-relativistic materials [7,8,11] and, in addition, on monolayer graphene (MG) [9,12,13]. The most interesting point of these contributions is that the new oscillatory character of the MCE as a function of the reciprocal magnetic field $1/H$ is analogous to the de Haas-van Alphen effect. Other interesting aspect of the oscillating magnetocaloric effect (OMCE) is its maximum value for the magnetic entropy change

even without phase transition (in high contrast to ordered materials that need a phase transition to maximize the MCE). In addition, deeper analysis of the OMCE on graphenes shows that the temperature in which the OMCE peaks are almost 100 times higher than non-relativistic diamagnetic materials (Gold, for instance). The OMCE also provides additional information about the system, like metrology of fundamental physical constants [12]. Thus, OMCE has interesting features that deserve to be further and deeper investigation; and graphenes are materials with the highest potential to show peculiar behaviors, mainly due to its relativistic features. Following this idea, the magnetocaloric properties of graphenes biased by an electric field were deeply investigated [15], but a drawback appeared: Joule heating is quite bigger than the magnetocaloric magnitude. The aim of this effort is therefore to overcome this problem. A straightforward idea is to consider a bilayer graphene with gate voltage applied between those two layers, avoiding the Joule heating.

Indeed, bilayer graphene (BG) is of special interest. From the theoretical viewpoint, BG is attractive due to the fact that electrons are described by an unusual wave equation, that has fourth-order derivatives, in contrast to the Schrödinger and Dirac equations [15]. In addition, an applied gate potential, perpendicular to the carbon planes, opens a gap in the energy spectrum of BG [16]. On the other hand, from the applied point of view, BG and MG-based devices may become promising materials for modern nanoelectronics [15], due to, for instance, an anomalous growth of the thermopower [17], unusual quantum Hall effect [18], exotic electrodynamic effects [19], giant magneto-optical [20] and thermomagnetic [21] effects.

In the present paper, thus, we deeply analyzed the OMCE in a BG and described the physical mechanisms that rule the oscillatory behavior of the magnetocaloric potentials as a function of

* Corresponding author.

E-mail address: zaur0102@gmail.com (Z.Z. Alisultanov).

temperature, reciprocal magnetic field and gate potential. We then found on BG a solution to observe a genuine OMCE, biased by an electric field, and without the inconvenience of Joule heating.

2. Bilayer graphene: electronic spectrum and Landau levels

For this effort, we will consider only the simplest theory of electronic spectrum and Landau levels – but, as will be clear further in the text, this procedure is enough to obtain the desired OMCE features of BG. The unit cell of BG has four carbon atoms and, consider the so-called AB stacking, the effective Hamiltonian can be written as [15]

$$\mathcal{H} = \begin{pmatrix} U & t_1^* & t_{\perp} & 0 \\ t_1 & U & 0 & 0 \\ t_{\perp} & 0 & -U & t_1 \\ 0 & 0 & t_1^* & -U \end{pmatrix} \quad (1)$$

where

$$\frac{t_{\perp}}{t} = e^{iq_x a} + e^{-iq_x a/2} e^{iq_y \sqrt{3}a/2} + e^{-iq_x a/2} e^{-iq_y \sqrt{3}a/2} \quad (2)$$

and t is the nearest-neighbor hopping energy (hopping between different sub-lattices in a monolayer). In addition, q represents the two-dimensional wave vector of electrons, a is the carbon-carbon distance in a monolayer, t_{\perp} is the hopping energy between two layers, i.e. hopping energy between A_1 and A_2 atoms (A_1 and B_1 are sub-lattices of the first layer, while A_2 and B_2 are sub-lattices of the second layer), and, finally, U is the gate potential, that, on its turn, tune the gap. Acceptable values for t and t_{\perp} are shown in Table 1, and, for the present effort, we will consider $t_{\perp} = 0.4$ eV and $t = 2.8$ eV.

The next step is to consider the region of momentum space in the vicinity of Dirac point and, in this region, the energy spectrum of the MG has a simple form:

$$\frac{t_1}{t} = \pm \frac{3}{2} a | \vec{k} | \quad (3)$$

where $\vec{k} = \vec{q} - \vec{Q}$ and $\vec{Q} = (2\pi/3a, 2\pi/3\sqrt{3}a)$ is the wave vector corresponding to the Dirac point. Above, \pm represents conduction ('+') and valence ('-') bands. Thus, the Hamiltonian on Eq. (1) gives the energy spectrum:

$$\epsilon(U, k) = \nu_b \left\{ v_F^2 h^2 k^2 + \frac{t_{\perp}^2}{2} + U^2 + \nu_{sb} \left[(t_{\perp}^2 + 4U^2) v_F^2 h^2 k^2 + \frac{t_{\perp}^4}{4} \right]^{1/2} \right\}^{1/2} \quad (4)$$

where $v_F = 3ta/2h = 10^8$ cm/s is the Fermi velocity of electrons; $\nu_b = \pm 1$ is a band index ($\nu_b = +1$ corresponds to conduction band and $\nu_b = -1$ corresponds to valence band); $\nu_{sb} = \pm 1$ is a sub-band index ($\nu_{sb} = +1$ corresponds to the sub-band I and $\nu_{sb} = -1$ corresponds to the sub-band II). Application of a voltage opens a gap

$$\Delta = \left[\frac{t_{\perp}^2 U^2}{t_{\perp}^2 + U^2} \right]^{1/2} \quad (5)$$

Table 1
Values of hopping energies.

Reference	t (eV)	t_{\perp} (eV)
Experiment [22] (IR spec.)	–	0.378(5)
Experiment [23] (IR spec.)	3.2(3)	0.381(3)
Experiment [24] (IR spec.)	3.0	0.40
Experiment [25] (Raman)	2.9	0.30
Experiment [26] (Raman)	3.0	0.35
Theory [27] (ab initio)	2.6	0.34
Theory [28] (ab initio)	3.4013	0.3963

in the energy spectrum, as can be seen in Fig. 1; and it is reasonable, since an external electric field breaks the symmetry between first and second layers.

To study the magnetic oscillations, we considered the quantization rule in the Lifshitz–Onsager form [29], since it has been successfully used in recent papers [30–32]. For a 2D electrons gas, the area enclosed by an electron trajectory in the momentum space is [29,33]

$$A(\epsilon) = \frac{2\pi h e H}{c} (n + \gamma_{\sigma}) \quad (6)$$

where $n = 0, 1, 2, \dots$,

$$\gamma_{\sigma} = \gamma + \frac{m(\mu)}{2m} \sigma, \quad (7)$$

$$m(\epsilon) = \frac{1}{2\pi} \frac{dA(\epsilon)}{d\epsilon} \quad (8)$$

is the electron cyclotron mass [33], m is the electron mass, $\sigma = \pm 1$ and, finally, $\gamma = 1/2$ for non-relativistic gas and $\gamma = 0$ for graphenes. In the present effort, the Zeeman splitting of the Landau levels is ignored, i.e., we assume that $m(\mu)/m = 0$ [34]. Considering $A(\epsilon) = \pi R^2(\epsilon)$, where $R(\epsilon)$ is the radius of an electron trajectory with constant energy, we can write therefore:

$$A(\epsilon) = \frac{\pi}{v_F^2} \left\{ \epsilon^2 + U^2 + [(4U^2 + t_{\perp}^2)\epsilon^2 - U^2 t_{\perp}^2]^{1/2} \right\} \quad (9)$$

Finally, the Landau levels can be derived from Eqs. (4) and (6); and reads then as

$$\epsilon_n = \left\{ \frac{2heH\nu_F^2}{c} + \frac{t_{\perp}^2}{2} + U^2 - \left[\left(t_{\perp}^2 + 4U^2 \right) \frac{2heH\nu_F^2}{c} + \frac{t_{\perp}^4}{4} \right]^{1/2} \right\}^{1/2} \quad (10)$$

It is important to stress that the limit in which t_{\perp} and U tends to zero, Eq. (9) reads as $A(\epsilon) = \pi\epsilon^2/v_F^2$ and Eq. (10) reads as $\epsilon_n = \sqrt{2heHn/c}$. Note these are the results for monolayer graphene [15] and the present evaluation can successfully recover those.

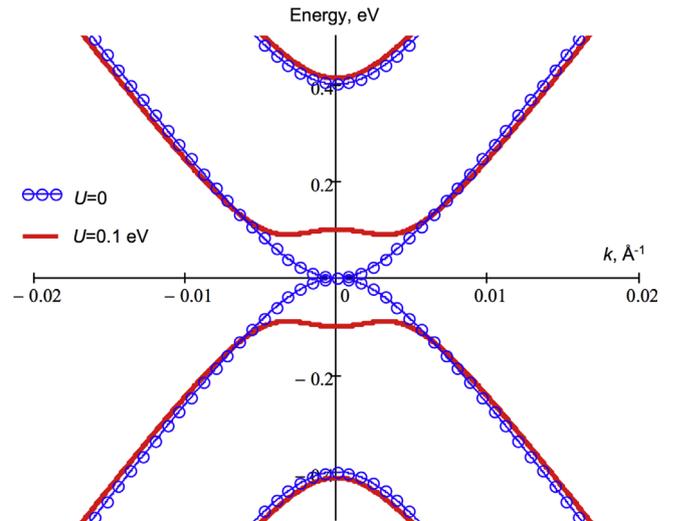


Fig. 1. (Color online) Energy spectrum of a bilayer graphene, where U is the gate potential between these two layers.

3. Magnetocaloric potentials

3.1. Magnetic entropy change

The MCE can be measured and estimated from the isothermal process and, for this case, the magnetic entropy change $\Delta S = S(T, H) - S(T, 0)$ is the quantity to be evaluated. Thus, we need to obtain the entropy of the system as a function of external parameters, like field and temperature. However, since the entropy of the system can be obtained from the grand potential Ω , i.e., $S = -\partial\Omega/\partial T$, we need first to evaluate the grand canonical potential, that reads as (see Appendix):

$$\Omega = \Omega(T, H) + \Omega(T, 0) \quad (11)$$

Note it has two contributions: one depends on the applied magnetic field and the other does not. Therefore, these contributions to the grand canonical potential per graphene area read as

$$\Omega(T, 0) = \Omega_0 = -\frac{1}{(\pi h)^2} \int_0^\infty A(\epsilon) f(\epsilon - \mu) d\epsilon \quad (12)$$

and

$$\Omega(T, H) = \Omega_H = \frac{2m(\epsilon_F)\omega_c k_B T}{\pi h} \sum_{k=1}^{\infty} \frac{1}{k} \frac{\cos\left[k \frac{A(\epsilon_F)c}{heH}\right]}{\sinh x_k} \quad (13)$$

where

$$x_k = x_k(T) = k \frac{2\pi^2 k_B T}{h\omega_c}, \quad (14)$$

$$\omega_c = \frac{eH}{m(\epsilon_F)c} \quad (15)$$

and $f(\epsilon - \mu)$ is the Fermi-Dirac distribution.

From the above, the field dependent part of the entropy (per graphene area) is

$$S_H(T) = \frac{k_B H}{\pi \Phi_0} \sum_{k=1}^{\infty} \frac{T(x_k)}{k} \cos\left[k \frac{A(\epsilon_F)c}{heH}\right] \quad (16)$$

where

$$T(y) = \frac{yL(y)}{\sinh y} \quad (17)$$

and $L(x_k) = \coth x_k - 1/x_k$ is the Langevin function. Above, Φ_0 is the magnetic flux quantum.

The zero-field entropy, per graphene area, can be determined as

$$S_0(T) = -\frac{\partial\Omega(T, 0)}{\partial T} \quad (18)$$

or

$$S_0(T) = \frac{1}{4\pi^2 h^2 k_B T^2} \int_0^\infty \frac{\epsilon - \mu}{\cosh^2\left[\frac{\epsilon - \mu}{2k_B T}\right]} A(\epsilon) d\epsilon \quad (19)$$

The integrand above is significantly different from zero only in the vicinity of $\epsilon \approx \mu$ and therefore it is true to consider $A(\epsilon) \approx A(\mu) + 2\pi m(\mu)(\epsilon - \mu)$. The zero-field contribution to the entropy (per graphene area) reads then as

$$S_0(T) \approx \frac{2\pi m(\epsilon_F)k_B^2 T}{3h^2} \quad (20)$$

To the above equation, we are also considering the low temperature regime ($\epsilon_F \gg k_B T$ and, consequently, $\mu \approx \epsilon_F$), as done for Eq. (16).

This section focus on the magnetic entropy change $\Delta S = S(T, H) - S(T, 0) = -S_H(T)$, that depends thus on the temperature T , magnetic field H and gate potential U ; and due not to the simple dependence of ΔS on these quantities, some pictures are welcome. Thus, the thermal dependence of the magnetic entropy change of a BG

is presented in Fig. 2; and, for comparison, the MG result, from Ref. [9], is also presented. Note, as mentioned before, the maximum on this quantity even without phase transition; and it is worth to note this maximum occurs at $T(y) = 1.6$ [9]. It is important to stress we considered only $k=1$ term on Eq. (16), as in Refs. [7–14]

This behavior of the entropy change can be qualitatively explained [14,35]: as long as the thermal energy is smaller than the distance between Landau levels, the kinetic energy of the electrons is unchangeable. A small increasing of temperature leads only to disordering of magnetic moments and, as a consequence, the magnetic entropy increases. For the case in which the thermal energy is greater than the distance between the Landau levels, the electrons can leave this level and move onto another one, i.e., the thermal energy is mainly spent to change the kinetic energy of electrons and, to a lesser extent, to violate the mutual orientation of the magnetic moments. In this case, the magnetic order of system restores, i.e., the magnetic part of entropy decreases.

The magnetic field dependence was also evaluated and presented in Fig. 3. Note it oscillates as a function of reciprocal magnetic field; and the mechanism that promotes these oscillations have the same nature as the de-Haas van Alphen effect. It is easy to understand that

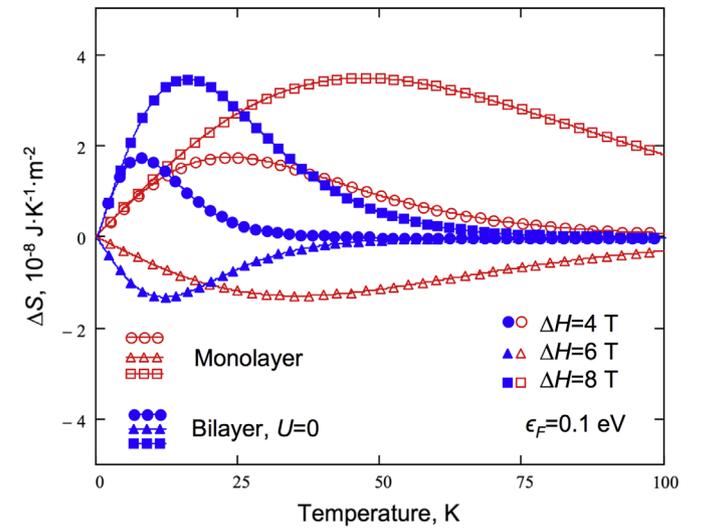


Fig. 2. (Color online) Thermal dependence of the magnetic entropy change for BG and MG, and several values of magnetic field change.

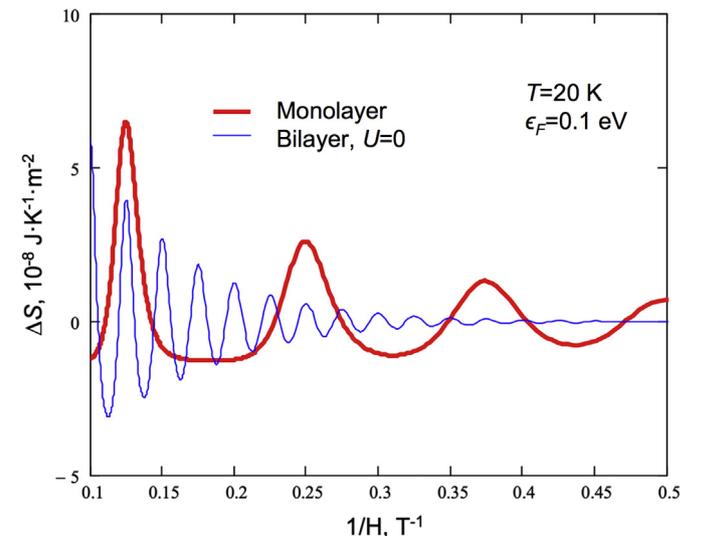


Fig. 3. (Color online) Magnetic entropy change of BG and MG as a function of reciprocal magnetic field.

the period of oscillations is determined as

$$\Delta\left(\frac{1}{H}\right) = \frac{\pi h e}{A(\epsilon_F) c} \quad (21)$$

3.2. Adiabatic temperature change

The adiabatic temperature change is also a magnetocaloric potential and thus deserves attention, in a similar fashion we performed above to magnetic entropy change. Indeed, oscillating adiabatic temperature change in 2D and 3D systems with a parabolic spectrum was already studied [8]. Considering the adiabatic condition $S(T_f, 0) = S(T_f, H)$ it is possible to write

$$\Delta T = T_f - T_i = -6T_f \sum_{k=1}^{\infty} \frac{L[x_k(T_f)]}{\sinh[x_k(T_f)]} \cos\left[k \frac{A(\epsilon_F) c}{h e H}\right] \quad (22)$$

It is easy to understand that, in this case, we have a self-consistent problem, since $T_f = T_i + \Delta T$, where T_i is the temperature of the system without applied magnetic field. Taking advantage of the interaction method, it is possible to write

$$\Delta T \approx -6(T + \Delta T_i) \sum_{k=1}^{\infty} \frac{L[x_k(T + \Delta T_i)]}{\sinh[x_k(T + \Delta T_i)]} \cos\left[k \frac{A(\epsilon_F) c}{h e H}\right] \quad (23)$$

where ΔT_i is determined from Eq. (22) with $T_f = T_i$.

Oscillations on the adiabatic temperature change of BG (Eq. (23)) are shown in Fig. 4. It should be noted that these oscillations are large in magnitude, in comparison, for instance, with other diamagnetic systems with the parabolic spectrum [8]; and, in fact, these values are comparable in magnitude with magneto-ordered systems. From Fig. 4 it is possible to see that application of gate potential leads to decrease of temperature change. It is a consequence to the fact that the opened band gap in the energy spectrum increases the effective mass of electrons in BG.

This adiabatic temperature change can be understood as follows: the temperature of the system is determined by the entropy that, on its turn, for a given energy, depends on either the concentration of energy states in the vicinity of this energy (Boltzmann thermodynamics) or the total number of states with the energy smaller than this energy (Gibbs thermodynamics). A change in these energy states changes consequently the temperature of the system; and therefore, under an applied magnetic field, due to the appearance of the Landau levels, the temperature of the system obviously changes. It is then worth to note that as greater the distance between these

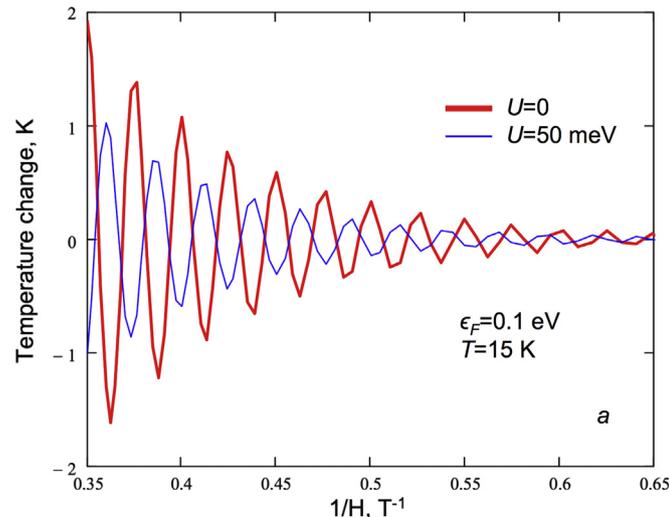


Fig. 4. (Color online) Adiabatic temperature change as a function of reciprocal magnetic field for BG.

levels is smaller the number of the levels and therefore the change in entropy increases. Taking into account that a distance between Landau levels is much larger in graphene than in non-relativistic materials, our result becomes clear.

3.3. Gate potential change

An interesting effect is the oscillation on the magnetic entropy change as a function of the gate potential U , as can be seen in Fig. 5. It is simple to understand, since the Landau levels (responsible for the thermodynamic oscillations), are U dependent. In other words, $A(\epsilon_F)$ depends on U . The adiabatic temperature change as a function of the gate potential is also shown – see Fig. 6.

Note this effect does not depend on electric moment polarization, but, on the contrary, it changes the Landau structure and then the caloric potentials. The isothermal effect is ruled by a change in entropy: $\Delta S = S(T, H, U) - S(T, H, 0)$; and the adiabatic effect is ruled by

$$S_0(T, U = 0) + S_H(T, U = 0) = S_{H=0}(T_f, U) + S_H(T_f, U) \quad (24)$$

The oscillating electrocaloric effect (OCE) in monolayer graphene in the longitudinal electric field was studied in Refs. [10,12]. Worth

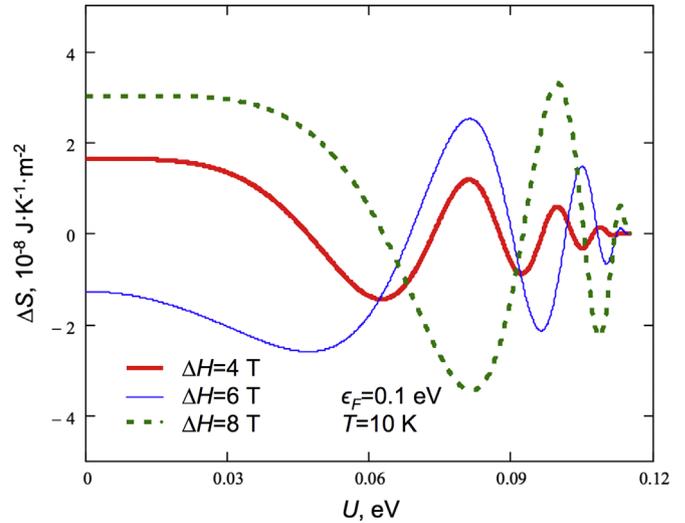


Fig. 5. (Color online) Energy spectrum of a bilayer graphene, where U is the gate potential between these two layers.

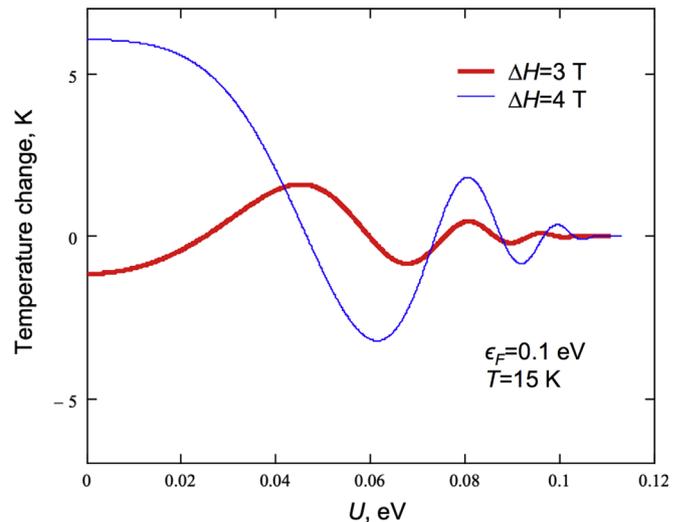


Fig. 6. (Color online) Adiabatic temperature change as a function of gate potential U .

to note: Joule heating on MG is generated under longitudinal electric field, and this effect is much bigger than the OECE. Thus, the experimental observation of OECE on MG under longitudinal electric field is a challenging task; and even the magnetocaloric potentials biased by an electric field. In contrast, the Joule heat is not generated in the BG in a transverse field – present work. Thus, the present work is a solution to overcome that problem.

4. Conclusions

Along this paper we discuss the oscillatory character of the thermodynamic potentials of bilayer graphenes, ruled by a gate potential between those two layers. The magnetic entropy change and the adiabatic temperature change were analytically obtained and we observe the oscillatory character of these two quantities as a function of the reciprocal magnetic field $1/H$. It is worth to note the amplitude of the adiabatic entropy change is comparable to magnetic ordered materials. In a similar fashion, we also describe the oscillating electrocaloric effect, due to a change on the gate potential between those two layers.

Finally, we stress that these results are similar to those described for monolayer graphene (MG) [9], however, the present contribution was able to overcome a strong drawback previously reported [9,13]: the electric field applied on MG promotes Joule heating and then a deeper damping of the caloric effect. At the present paper the Joule heating problem does not exist, since the gate potential is applied between the two layers. We are convinced that the present effort is an important contribution to the knowledge of the oscillating thermodynamic properties.

Acknowledgments

Authors thank L.A. Falkovsky for useful discussion on the paper. M.S. Reis thanks FAPERJ, CAPES, CNPq and Proppi-UFF for the financial support. Z.Z.A. emphasizes that the reported study was partially supported by Project no. 3.1262.2014K of Ministry of Education and Science of Russia in scientific research and RFBR research Project no. 15-02-03311a.

Appendix A. Evaluation of Ω_H

The general expression for the field dependent thermodynamic potential (per graphene area), is

$$\Omega_H = -\frac{2eHk_B T}{\pi hc} \sum_n \ln \left[1 + \exp \left(\frac{\mu - \epsilon_n}{k_B T} \right) \right] \quad (\text{A.1})$$

where T is the temperature, μ is the chemical potential and k_B is the Boltzmann constant. The present effort considers the electron doping case and therefore $\mu > 0$. Using the Poisson summation formula

$$\sum_{n=0}^{\infty} F(n) = \int_0^{\infty} F(x) dx + 2\text{Re} \left[\sum_{k=1}^{\infty} \int_0^{\infty} F(x) e^{i2\pi kx} dx \right] \quad (\text{A.2})$$

we obtain

$$\Omega_H = -\frac{2eHk_B T}{\pi hc} \left\{ \int_0^{\infty} \ln \left[1 + \exp \left(\frac{\mu - \epsilon_x}{k_B T} \right) \right] dx + 2\text{Re} \left[\sum_{k=1}^{\infty} \int_0^{\infty} \ln \left[1 + \exp \left(\frac{\mu - \epsilon_x}{k_B T} \right) \right] e^{i2\pi kx} dx \right] \right\} \quad (\text{A.3})$$

Integrating by parts and using Eq. (6), we obtain

$$\Omega_H = -\frac{1}{(\pi h)^2} \int_0^{\infty} A(\epsilon) f(\epsilon - \mu) d\epsilon - \frac{2eH}{\pi^2 hc} \text{Re} \left[\sum_{k=1}^{\infty} \frac{1}{ik} \int_0^{\infty} f(\epsilon - \mu) \exp \left(\frac{ikcA(\epsilon)}{heH} \right) d\epsilon \right] \quad (\text{A.4})$$

The energies $\epsilon \approx \mu$ are important for the magnetic oscillations and therefore we expand the function $A(\epsilon)$ in the vicinity of μ , i.e. $A(\epsilon) \approx A(\mu) + 2\pi m(\mu)(\epsilon - \mu)$. Then, integrating and taking into account that $\mu \ll k_B T$ we obtained the following expression for field-dependent part of thermodynamic potential:

$$\Omega_H = \frac{2m(\mu)\omega_c k_B T}{\pi h} \sum_{k=1}^{\infty} \frac{1}{k} \frac{\cos(kA(\mu)c/heH)}{\sinh(k2\pi^2 k_B T/h\omega_c)} \quad (\text{A.5})$$

where

$$\omega_c = \frac{eH}{m(\epsilon_F)c} \quad (\text{A.6})$$

To obtain Eq. (A.5) we used the relation:

$$\int_{-\infty}^{\infty} \frac{e^{iaz}}{e^z + 1} dz = -\frac{i\pi}{\sinh(a\pi)} \quad (\text{A.7})$$

References

- [1] A.M. Tishin, Y.L. Spichkin, *The Magnetocaloric Effect and its Applications*, Inst. of Physics Pub. Inc., Cornwall, 2003.
- [2] M. Reis, *Fundamentals of Magnetism*, Elsevier, New York, 2010.
- [3] A. Andreev, K. Belov, S. Nikitin, A. Tishin, *Sov. Phys. Uspekhi* 32 (1989) 649.
- [4] N. de Oliveira, P. von Ranke, *Phys. Rep.* 489 (2010) 89.
- [5] M.-H. Phan, S.-C. Yu, *J. Magn. Magn. Mater.* 308 (2007) 325.
- [6] N. Bebenin, R. Zainullina, V. Ustinov, *J. Appl. Phys.* 113 (2013) 073907.
- [7] M. Reis, *Appl. Phys. Lett.* 99 (2011) 052511.
- [8] M. Reis, *Solid State Commun.* 152 (2012) 921.
- [9] M. Reis, *Appl. Phys. Lett.* 101 (2012) 222405.
- [10] M. Reis, *Solid State Commun.* 161 (2013) 19.
- [11] M. Reis, *J. Appl. Phys.* 113 (2013) 243901.
- [12] M. Reis, *Phys. Lett. A* 378 (2014) 918.
- [13] L. Paixão, Z. Alisultanov, M. Reis, *J. Magn. Magn. Mater.* 368 (2014) 374.
- [14] Z. Alisultanov, *J. Appl. Phys.* 115 (2014) 113913.
- [15] M. Katsnelson, *Graphene: Carbon in Two Dimensions*, Cambridge University Press, New York, 2012.
- [16] E. Castro, K. Novoselov, S. Morozov, *Phys. Rev. Lett.* 99 (2007) 216802.
- [17] Z. Alisultanov, *JETP Lett.* 98 (2013) 111.
- [18] K. Novoselov, A. Geim, S. Morozov, D. Jiang, M. Katsnelson, I. Grigorieva, S. Dubonos, A. Firsov, *Nature* 438 (2005) 197.
- [19] M. Katsnelson, K. Novoselov, A. Geim, *Nat. Phys.* 2 (2006) 620.
- [20] L. Falkovsky, *Zh. Exp. Theor. Fiz.* 142 (2012) 1309.
- [21] I. LukOyanchuk, A. Varlamov, A. Kavokin, *Phys. Rev. Lett.* 107 (2011) 016601.
- [22] A. Kuzmenko, E. van Heumen, D. Marel, *Phys. Rev. B* 79 (2009) 115441.
- [23] A. Kuzmenko, I. Crassee, D. Marel, *Phys. Rev. B* 80 (2009) 165406.
- [24] L. Zhang, Z. Li, D. Basov, *Phys. Rev. B* 78 (2008) 235408.
- [25] L. Malard, J. Nilsson, D. Elias, *Phys. Rev. B* 76 (2007) 201401 (R).
- [26] L. Malard, J. Nilsson, D. Mafra, *Phys. Status Solidi b* 245 (2008) 2060.
- [27] H. Min, B. Sahu, S. Banerjee, A. MacDonald, *Phys. Rev. B* 75 (2007) 155115.
- [28] P. Gava, M. Lazzeri, A.M. Saitta, F. Mauri, *Phys. Rev. B* 79 (2009) 165431.
- [29] I. Lifshitz, M. Kaganov, *Sov. Phys. Uspekhi* 2 (1959) 831.
- [30] A. Ozerin, L. Falkovsky, *Phys. Rev. B* 85 (2012) 205143.
- [31] Z. Alisultanov, *Physica B* 438 (2014) 41.
- [32] Z. Alisultanov, *JETP Lett.* 99 (2014) 232.
- [33] I.M. Lifshitz, A.M. Kosevich, *Zh. Eksp. Teor. Fiz.* 29 (1955) 730.
- [34] I. Luk Oyanchuk, Y. Kopelevich, *Phys. Rev. Lett.* 93 (2004) 166402.
- [35] Z.Z. Alisultanov, *J. Exp. Theor. Phys.* 119 (2) (2014) 300–310, <http://dx.doi.org/10.1134/S1063776114070012>.