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Oscillating adiabatic temperature change of diamagnetic materials

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ABSTRACT

The fundamental model to describe a diamagnetic material is an electron gas; and a huge applied magnetic field promotes a degeneracy, named Landau levels. Oscillations on the thermodynamic quantities are found when the Landau levels cross the Fermi energy of the non-perturbed gas at low temperatures regime ($\varepsilon_F \gg k_BT$). The adiabatic temperature change ΔT characterizes the magnetocaloric properties and here this quantity is described for the system above mentioned. We show that ΔT oscillations are possible to be found in Gold in standard values of temperature and magnetic field; *c.a.*2 K and 8.5 T. These oscillations are around zero, i.e., both, normal and inverse magnetocaloric effect arises and a possible application of this effect is in magnetic field sensor, since this system is sensible to 0.8 mT with a thermal response of 50 mK (at 2.56 K).

1. Introduction

The magnetocaloric properties of materials have been extensively explored by the scientific community for the last decades [1], due to several possible applications; from low temperatures (mK scale), e.g., adiabatic demagnetization refrigerator [2] up to room temperature, e.g., air-conditioners and industrial/domestic fridges [3,4]. Magnetic materials for these applications have been the core of research to improve magnetocaloric properties, however, efforts are restricted to ferro-, antiferro-, ferri- and even para-magnetic materials [1]; while the diamagnetic ones were never discussed, except recently [5].

Due to the crossing of the Landau levels through the Fermi energy ε_F of an electron gas (this last models a diamagnetic material), oscillations on the thermodynamic properties are found; as the well known de Haas–van Alphen effect [6]. Following this fact, we have recently described in details the isothermal magnetocaloric properties, i.e., the magnetic entropy change, of diamagnetic materials [5]. We found that this quantity only depends on the oscillating contribution to the entropy (the field dependent contribution that does not contain the oscillating term is null), and therefore it can be either inverse or normal, depending on the value of the applied magnetic field change. We understand normal as the negative magnetic entropy change; and inverse as the positive one.

Those results open doors for new applications as described in Ref. [5], namely in adiabatic demagnetization refrigerators and magnetic field sensors. This last proposal is based on the fact that this oscillating effect is sensible to *c.a.*1 mT with a huge magnetic field on the background (*c.a.*10 T), at low temperatures, *c.a.*1 K.

Since the previous contribution dealt with the magnetic entropy change, here, to complete the description of the magnetocaloric properties of diamagnetic materials, we describe the adiabatic temperature change. Next section a brief survey of the model is presented and then the adiabatic temperature change. Finally, by the end of the paper, a potential application in magnetic field sensor is further discussed.

2. Brief survey

Let us consider an electron gas under an applied magnetic field, following the conditions: $\varepsilon_F \gg k_BT$ and μ_BB ranges from k_{BT} up to ε_F. The magnetic entropy $\mathcal{S}(T,B)$ depends on the *grand* canonical partition function $Z(T,B)$ through

$$\mathcal{S}(T,B) = k_B \frac{\partial}{\partial T}[T \ln Z(T,B)], \tag{1}$$

where

$$\ln Z = \int_0^\infty d\varepsilon\, g(\varepsilon) \ln(1+ze^{-\varepsilon/k_BT}), \tag{2}$$

$z = e^{\mu/k_BT}$ is the fugacity and $g(\varepsilon)$ is the density of states, given by [6]

$$g(\varepsilon) = \frac{3}{2}N\varepsilon_F^{-3/2}\left\{\varepsilon^{1/2} + (\mu_BB)^{1/2}\sum_{l=1}^{+\infty}\frac{(-1)^l}{l^{1/2}}\cos\left(\frac{l\pi}{\mu_BB}\epsilon - \frac{\pi}{4}\right)\right\}$$
$$= g_0(\varepsilon) + g_B(\varepsilon). \tag{3}$$

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Note therefore that there are two contributions to the logarithm of the *grand* partition function; one that does not depend on the magnetic field B and other that does, i.e.,

$$\ln Z = \ln Z_0 + \ln Z_B. \tag{4}$$

Thus, the magnetic entropy can be written as

$$\mathcal{S}(T,B) = S(T,0) + S_B(T,B). \tag{5}$$

The first term is the entropy of a free electron gas without applied magnetic field, known from textbooks [6]

$$\frac{\mathcal{S}(T,0)}{Nk_B} = \frac{\pi^2}{2}y, \tag{6}$$

where $y = k_B T/\varepsilon_F$. Now, we need to consider the field dependent term; that, after some calculation (further details in Ref. [5]), it is possible to see that there are two contributions

$$\ln Z_B = \ln Z_B^{no} + \ln Z_B^{o}, \tag{7}$$

where the first is

$$\ln Z_B^{no} = -\frac{N}{k_B T}\left(\frac{\mu_B B}{\varepsilon_F}\right)^{3/2}(0.26\varepsilon_F + 0.09\mu_B B), \tag{8}$$

and has a non-oscillatory character and the second contribution is

$$\ln Z_B^{o} = -\frac{3}{2}N\left(\frac{\mu_B B}{\varepsilon_F}\right)^{3/2}\sum_{l=1}^{+\infty}\frac{(-1)^l}{l^{3/2}}\cos\left(\frac{l\pi}{\mu_B B}\varepsilon_F - \frac{\pi}{4}\right)\frac{1}{\sinh(x_l)}, \tag{9}$$

and has an oscillatory character. In addition

$$x_l = l\pi^2\frac{k_B T}{\mu_B B}. \tag{10}$$

From Eq. (7), we see that the field dependent entropy has two contributions

$$S_B(T,B) = S_B^{no}(T,B) + S_B^{o}(T,B) \tag{11}$$

however, the non-oscillatory term is null [5]

$$S_B^{no}(T,B) = 0 \tag{12}$$

and thus

$$\frac{S_B^{o}(T,B)}{Nk_B} = -\frac{3}{2}\left(\frac{1}{n+1/4}\right)^{3/2}\cos(n\pi)\,\mathcal{T}(x), \tag{13}$$

where

$$\mathcal{T}(x) = [\sinh(x)]^{-1}xL(x), \tag{14}$$

and $L(x)$ is the Langevin function

$$L(x) = \coth(x) - \frac{1}{x}. \tag{15}$$

In addition

$$x = \pi^2 y\left(n + \frac{1}{4}\right) \tag{16}$$

and

$$n = \frac{\varepsilon_F}{\mu_B B} - \frac{1}{4}. \tag{17}$$

Note, from Eq. (9)–(13) we considered only the $l=1$ term, as done before [5,6]; but it is easy to understand: the hyperbolic sine in the denominator of Eq. (14), damping the entropy for low values of l (≥ 2).

3. Adiabatic temperature change

From Eq. (5), (6), (11)–(13), it is possible to obtain the adiabatic temperature change of this diamagnetic system. The condition is imposed considering that the entropy of the system at T_0 and zero field is the same of the system at T_B and under an applied magnetic field B. Thus

$$\mathcal{S}(T_0,0) = \mathcal{S}(T_B,B), \tag{18}$$

and therefore

$$\frac{\pi^2}{2}y_0 = \frac{\pi^2}{2}y_B - \frac{3}{2}\left(\frac{1}{n+1/4}\right)^{3/2}\cos(n\pi)\mathcal{T}(x_B), \tag{19}$$

where $y_0 = y(T_0)$ and $x_B = x(y(T_B))$. This condition gives a relation among T_0 and T_B and then it is possible to write the adiabatic temperature change $\Delta T = T_B - T_0$. However, the function $T_0(T_B)$ from Eq. (19) is not simple and an approximation is needed. In this sense, we can consider other constraint to the system: $x \ll 1$. It is reasonable, because $y \ll 1$; and we only need to ensure that n ranges from the unity up to the order of $1/y$. Following this condition, Eq. (14) resumes as

$$\mathcal{T}(x) \approx \frac{x}{3}, \tag{20}$$

and therefore Eq. (19) can be rewritten as

$$y_0 = y_B\left[1 - \frac{\cos(n\pi)}{\sqrt{n+1/4}}\right]. \tag{21}$$

The above result can now be useful. Considering the definition of adiabatic temperature change $\Delta T = T_B - T_0$, we can write our final result

$$\Delta T = T_0\left[\frac{\cos(n\pi)}{\sqrt{n+1/4} - \cos(n\pi)}\right]. \tag{22}$$

Note if $\sqrt{n+1/4} = \cos(n\pi)$ the adiabatic temperature changes diverge; and it occur for $n = 1/4$, that correspond to $B = 2\varepsilon_F/\mu_B$, that no longer fulfill the condition stated above, in which the magnetic field must be up to the order of the Fermi energy. On the other hand, for $n \gg 1$, Eq. (22) resumes as

$$\Delta T = T_0\left[\frac{\cos(n\pi)}{\sqrt{n}}\right]. \tag{23}$$

Thus, for n even, the adiabatic temperature change is positive, i.e., $T_B > T_0$, and the system warms up due to an applied magnetic field. On the other hand, for n odd: $T_B < T_0$, and the system cools down due to an applied magnetic field. Of course, for half integer values of n, the adiabatic temperature change is zero.

Fig. 1 presents the oscillatory behavior found for the adiabatic temperature change ΔT (Eq. (22)), as well as the approximation for large values of n (Eq. (23)). Note n is inversely proportional to

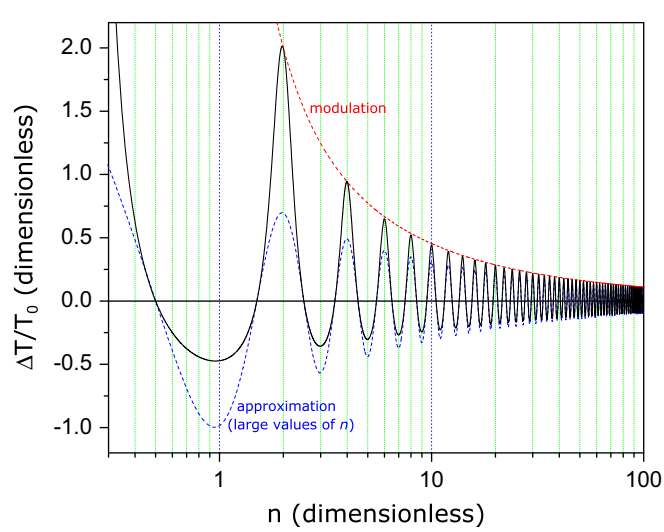

Fig. 1. (Color online) Reduced adiabatic temperature change $\Delta T/T_0$ as a function of the n (a function of the inverse magnetic field B).

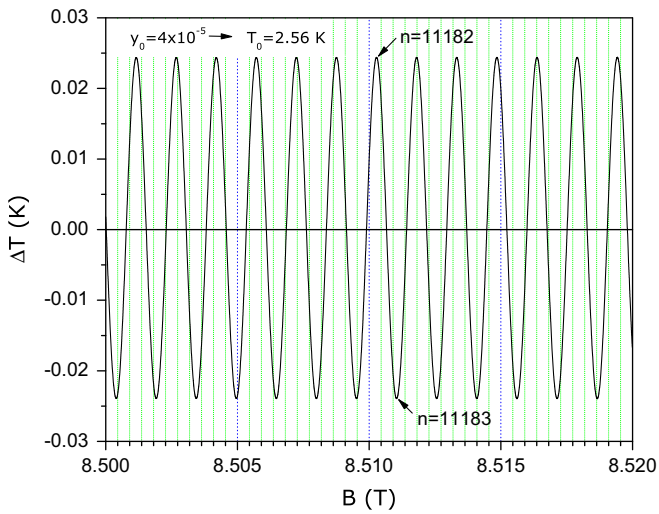

Fig. 2. (Color online) Adiabatic temperature change in real units, i.e., Kelvin, as function of the applied magnetic field B, in Tesla. This example is for Gold ($\varepsilon_F = 5.51$ eV), under accessible temperature (2.56 K) and magnetic field (*c.a.*8.5 T) ranges.

B (Eq. (17)). This figure also contains the modulation of the oscillations found, obtained considering any even number for n into cosines of Eq. (22).

However, it is hard to see the broadness of the effect (in spite of its curiosity), working with dimensionless quantities. In this sense, Fig. 2 presents the prediction of this effect for Gold ($\varepsilon_F = 5.51$ eV), for $T_0 = 2.56$ K and around 8.5 T of magnetic field change. Note therefore that this effect is comfortably visible within that range of magnetic field and temperature, reasonable for standard laboratories. The chosen value of T_0 temperature is easy to understand. From Ref. [5], the maximum magnetic entropy change of this system was found around $y = 2 \times 10^{-5}$, that corresponds to 1.28 K; a temperature range not easy to work. However, the effect is still seen around $y = 4 \times 10^{-5}$, that corresponds to that $T_0 = 2.56$ K; easier temperature range to work.

4. Possible application

As proposed before [5], this system can be used as a magnetic field sensor; since it changes its temperature due to an applied magnetic field. Since it has an oscillatory behavior, the absolute value of the magnetic field is not possible to be achieved, but, of course, it senses a magnetic field change.

Considering Fig. 2, it is possible to see that Gold produces $|\Delta T(B(n = 11\,182)) - \Delta T(B(n = 11\,183))| = 50$ mK of change of the adiabatic temperature change (around 2.56 K), due to $|B(n = 11\,182) - B(n = 11\,183)| = 0.8$ mT of change of the magnetic field change (with a huge magnetic field on the background, *c.a.*8.5 T). This temperature change is easily measured with a cernox-like sensor, since its accuracy is around 5 mK for this range of temperature.

5. Conclusion

This paper describes the adiabatic temperature change of a diamagnetic material. Due to the crossing of the Landau levels through the Fermi level, oscillation appears; this is the well known de Haas–van Alphen effect, observed in magnetic measurements. Recently, we explored the influence of these oscillations on the magnetic entropy change of diamagnetic materials [5] and here we extended our evaluation to obtain the adiabatic temperature change. Oscillations indeed appear and can be explored to be useful in magnetic field sensors. As an example, for Gold, this effect makes possible the sensibility to 0.8 mT (around 2.56 K), to a thermal response of 50 mK (measurable with a cernox-like sensor).

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