Generalized entropic measures of quantum correlations

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Majorization
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$$\rho \prec \rho' \iff \sum_{j=1}^{i} p_j \leq \sum_{j=1}^{i} p'_j, \quad j = 1, \ldots, n - 1$$

$p_j, p'_j$ eigenvalues of $\rho, \rho'$ sorted in decreasing order
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$\left(\frac{1}{n}, \ldots, \frac{1}{n}\right) \prec p \prec (1, 0, \ldots, 0) \quad \forall \, p \quad \Rightarrow \quad \frac{1}{n} \prec \rho \prec \left|\Psi\right\rangle\left\langle\Psi\right| \quad \forall \, \rho$
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Relation with von Neumann entropy:

$$\rho \prec \rho' \Rightarrow S(\rho) \geq S(\rho')$$
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$$\rho \prec \rho' \Rightarrow S(\rho) \geq S(\rho') \quad \text{but} \quad S(\rho) \geq S(\rho') \not\Rightarrow \rho \prec \rho'$$
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Majorization stronger than basic entropic criterion
Majorization and generalized entropies

Majorization actually implies *universal* entropy increase:

\[ \rho < \rho' \Rightarrow S_f(\rho) \geq S_f(\rho') \land S_f(\rho) \]
Majorization and generalized entropies

Majorization actually implies \textit{universal} entropy increase:

\[ \rho \prec \rho' \implies S_f(\rho) \geq S_f(\rho') \quad \forall S_f(\rho) \]

for all generalized entropies \( S_f(\rho) = \text{Tr } f(\rho) \)

with \( f(\rho) \) an \textit{arbitrary} concave function satisfying \( f(0) = f(1) = 0 \)
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All generalized entropies \( S_f(\rho) = \text{Tr} \ f(\rho) \) satisfy \cite{RR-NC PRL 88 (2002)}:
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\[ S(\rho) = -\text{Tr} \, \rho \log \rho \text{ von Neumann entropy} \]
\[ S_2(\rho) = 2\text{Tr} \, (\rho - \rho^2) \text{ Linear entropy} \]
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\[ S_q(\rho) = \frac{1}{1-2^{1-q}} \text{Tr} \ (\rho - \rho^q) \]
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$S_q(\rho) = \frac{1}{1-q} \text{Tr} \ (\rho - \rho^q)$

$S_q(\rho) \rightarrow S_1(\rho) = S(\rho)$ for $q \rightarrow 1$
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Measurement and Majorization
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Initial quantum state $\rho$
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Measurement $M = \{P_k\}$, $\sum_k P_k = I$, $P_k P_{k'} = \delta_{kk'} P_k$
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Initial quantum state $\rho$

Measurement $M = \{P_k\}, \sum_k P_k = I, P_k P_{k'} = \delta_{kk'} P_k$

State after unread measurement:

$$\rho' = \sum_k p_k \rho_k = \sum_k P_k \rho P_k$$
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Fundamental property: $\rho' \prec \rho$
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Measurement $M = \{P_k\}, \sum_k P_k = I, P_k P_{k'} = \delta_{kk'} P_k$

State after unread measurement:

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Implies

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S_f(\rho') \geq S_f(\rho) \quad \forall S_f
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Increase of von Neumann entropy just a particular case
Information loss by unread measurement
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\[ I_f^M(\rho) = S_f(\rho') - S_f(\rho) \]
Information loss by unread measurement

\[ I^M_f(\rho) = S_f(\rho') - S_f(\rho) \]

with \( S_f(\rho) = \text{Tr} \ f(\rho) \) generalized entropic form

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RR-NC-LC PRA 82 (2010)

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RR-NC-LC PRA 82 (2010)

\( \rho' \prec \rho \) implies \( I_f^M(\rho) \geq 0 \forall S_f \) with \( I_f^M(\rho) = 0 \) iff \( \rho' = \rho \)
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Special cases:
Information loss by unread measurement

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\( \rho' \prec \rho \) implies \( I^M_f(\rho) \geq 0 \ \forall \ S_f \) with \( I^M_f(\rho) = 0 \) iff \( \rho' = \rho \)

Special cases:

**Von Neumann entropy** \((S_f(\rho) = -\text{Tr} \rho \log \rho)\):

\[ I^M_{1}(\rho) = \text{Tr} \rho (\log \rho - \log \rho') = S(\rho || \rho') \]
Information loss by unread measurement

\[ I^M_f(\rho) = S_f(\rho') - S_f(\rho) \]

with \( S_f(\rho) = \text{Tr}(f(\rho)) \) generalized entropic form

RR-NC-LC PRA 82 (2010)

\( \rho' \prec \rho \) implies \( I^M_f(\rho) \geq 0 \) \( \forall S_f \) with \( I^M_f(\rho) = 0 \) iff \( \rho' = \rho \)

Special cases:

**Von Neumann entropy** \( (S_f(\rho) = -\text{Tr}(\rho \log \rho)) \):

\[ I^M_1(\rho) = \text{Tr}(\rho(\log \rho - \log \rho')) = S(\rho\|\rho') \]

**Linear entropy** \( (S_f(\rho) = \text{Tr}(\rho - \rho^2)) \):

\[ I^M_2(\rho) = \text{Tr}(\rho^2 - \rho'^2) = \|\rho - \rho'\|^2 \]
Local measurement in bipartite system
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Initial state $\rho_{AB}$
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$M_B = \{ I_A \otimes P_j^B \}$
Local measurement in bipartite system

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State after unread measurement: $\rho'_{AB} = \sum_j p_j \rho_{A/j} \otimes P_j^B$
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$p_j = \text{Tr} (\rho_{AB} I_A \otimes P_j^B)$, $\rho_{A/j} \propto \text{Tr}_B (\rho_{AB} I_A \otimes P_j^B)$
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Information loss:

$I^M_B (\rho_{AB}) = S_f (\rho'_{AB}) - S_f (\rho_{AB})$
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Minimum Information loss due to unread local measurement:
Local measurement in bipartite system

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State after unread measurement:

$\rho'_{AB} = \sum_j p_j \rho_{A/j} \otimes P^B_j$

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Information loss:

$I_{MB}^M(\rho_{AB}) = S_f(\rho'_{AB}) - S_f(\rho_{AB})$

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$I^B_f(\rho_{AB}) = \min_{M_B} I^{M_B}_f(\rho_{AB})$

$I^B_f(\rho_{AB}) \geq 0$
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Minimum Information loss due to unread local measurement:

$I^B_f(\rho_{AB}) = \text{Min}_{M_B} I^{M_B}_f(\rho_{AB})$

$I^B_f(\rho_{AB}) \geq 0$ with $I^B_f(\rho_{AB}) = 0$ iff $\rho_{AB} = \sum_j p_j \rho_{A/j} \otimes P^B_j$

$I^B_f$ Generalized entropic measure of quantum correlations

RR NC LC PRA 82 (2010)
Pure states
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If $\rho^2_{AB} = \rho_{AB} \Rightarrow I^B_f(\rho_{AB}) = E_f(A, B)$
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If $\rho_{AB}^2 = \rho_{AB} \Rightarrow I_f^B(\rho_{AB}) = E_f(A, B)$

$E_f(A, B) = S_f(\rho_A) = S_f(\rho_B)$ generalized entanglement entropy

$\rho_A = \text{Tr}_B \rho_{AB}, \rho_B = \text{Tr}_A \rho_{AB}$
Pure states

If \( \rho_{AB}^2 = \rho_{AB} \Rightarrow I_f^B(\rho_{AB}) = E_f(A, B) \)

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\( \rho_A = \text{Tr}_B \rho_{AB}, \rho_B = \text{Tr}_A \rho_{AB} \)

Von Neumann case: \( E_f(A, B) = S(\rho_A) \) std. entangl. entropy
Pure states

If \( \rho_{AB}^2 = \rho_{AB} \) \( \Rightarrow \) \( I_f^B(\rho_{AB}) = E_f(A, B) \)

\( E_f(A, B) = S_f(\rho_A) = S_f(\rho_B) \) generalized entanglement entropy
\( \rho_A = \text{Tr}_B \rho_{AB}, \rho_B = \text{Tr}_A \rho_{AB} \)

Von Neumann case: \( E_f(A, B) = S(\rho_A) \) std. entangl. entropy
Linear entropy case: \( E_f(A, B) = C^2(A, B) = 2(1 - \text{Tr} \rho_A^2) \) (squared concurrence)
Pure states

If \( \rho_{AB}^2 = \rho_{AB} \) \( \Rightarrow I_f^B(\rho_{AB}) = E_f(A, B) \)

\[ E_f(A, B) = S_f(\rho_A) = S_f(\rho_B) \text{ generalized entanglement entropy} \]

\[ \rho_A = \text{Tr}_B \rho_{AB}, \rho_B = \text{Tr}_A \rho_{AB} \]

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(squared concurrence)

Universal least disturbing local measurement: Schmidt basis
Pure states

If $\rho_{AB}^2 = \rho_{AB} \Rightarrow I_f^B(\rho_{AB}) = E_f(A, B)$

$E_f(A, B) = S_f(\rho_A) = S_f(\rho_B)$ generalized entanglement entropy

$\rho_A = \text{Tr}_B \rho_{AB}, \rho_B = \text{Tr}_A \rho_{AB}$

Von Neumann case: $E_f(A, B) = S(\rho_A)$ std. entangl. entropy

Linear entropy case: $E_f(A, B) = C^2(A, B) = 2(1 - \text{Tr} \rho_A^2)$ (squared concurrence)

Universal least disturbing local measurement: Schmidt basis

$\rho_{AB} = |\Psi_{AB}\rangle \langle \Psi_{AB}|, |\Psi_{AB}\rangle = \sum_k \sqrt{p_k} |k_A\rangle |k_B\rangle \Rightarrow M_B = \{P_k^B\}$
Pure states

If \( \rho_{AB}^2 = \rho_{AB} \Rightarrow I_{f}^B(\rho_{AB}) = E_f(A, B) \)

\( E_f(A, B) = S_f(\rho_A) = S_f(\rho_B) \) generalized entanglement entropy

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\( \rho'_{AB} = \sum_k p_k P_k^A \otimes P_k^B \) least mixed state after local measurement
Mixed states
Mixed states
Mixed states

\[ I_f^B (\rho_{AB}) = 0 \iff \rho_{AB} \text{ classically correlated (from } B) \]
Mixed states

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Vanishes for the same states as the Quantum Discord
Mixed states

\[ I_f^B(\rho_{AB}) = 0 \iff \rho_{AB} \text{ classically correlated (from } B) \]

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\[ \rho_{AB} \text{ separable } \Rightarrow I_f^B(\rho_{AB}) = 0 \]
Mixed states

\[ I_f^B(\rho_{AB}) = 0 \iff \rho_{AB} \text{ classically correlated (from } B) \]

Vanishes for the same states as the Quantum Discord

\[ \rho_{AB} \text{ separable } \not\Rightarrow I_f^B(\rho_{AB}) = 0 \]

\[ (\rho_{AB} \text{ separable } \iff \rho_{AB} = \sum_\alpha q_\alpha \rho^\alpha_A \otimes \rho^\alpha_B, q_\alpha > 0) \]

Minimum of \( I_f^{MB} \) no longer universal in general
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Minimum of \( I_f^{MB} \) no longer universal in general

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Minimum of \( I_f^{MB} \) no longer universal in general

\( I_f^B(\rho_{AB}) \) not an upper bound to \( E_f(A, B) \) in general

Generalized entanglement of formation:

\[ E_f(A, B) = \text{Min} \sum_\alpha q_\alpha E_f(|\Psi^\alpha_{AB}\rangle) / \rho_{AB} = \sum_\alpha q_\alpha |\Psi^\alpha_{AB}\rangle \langle \Psi^\alpha_{AB}|, q_\alpha \geq 0 \]
Other local measurements
Other local measurements

\[ I_f^A(\rho_{AB}) = \operatorname{Min}_{M_A} I_f^{M_A}(\rho_{AB}) \quad M_A = \{ P_i^A \otimes I_B \} \]
Other local measurements

\[ I_f^A(\rho_{AB}) = \min_{M_A} I_f^{MA}(\rho_{AB}) \quad M_A = \{P_i^A \otimes I_B\} \]

\[ I_f^{AB}(\rho_{AB}) = \min_{M_{AB}} I_f^{M_{AB}}(\rho_{AB}) \quad M_{AB} = \{P_i^A \otimes P_j^B\} \]
Other local measurements

\[ I_A^f(\rho_{AB}) = \min_{M_A} I_{f}^{M_A}(\rho_{AB}) \quad M_A = \{P_i^A \otimes I_B\} \]

\[ I_{AB}^f(\rho_{AB}) = \min_{M_{AB}} I_{f}^{M_{AB}}(\rho_{AB}) \quad M_{AB} = \{P_i^A \otimes P_j^B\} \]

\[ I_{AB}^f(\rho_{AB}) \geq 0, \text{ with } I_{AB}^f(\rho_{AB}) = 0 \text{ iff } \rho_{AB} = \sum_{i,j} p_{ij} P_i^A \otimes P_j^B \]
Other local measurements

\[ I_f^A(\rho_{AB}) = \min_{M_A} I_f^{MA}(\rho_{AB}) \quad M_A = \{P_i^A \otimes I_B\} \]

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classically correlated state
Other local measurements

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classically correlated state

Pure states: \[ I^A_f = I^{AB}_f = I^B_f = E_f(A, B) \]
Other local measurements

\[ I^A_f(\rho_{AB}) = \underset{M_A}{\text{Min}} I^M_A(\rho_{AB}) \quad M_A = \{ P_i^A \otimes I_B \} \]

\[ I^{AB}_f(\rho_{AB}) = \underset{M_{AB}}{\text{Min}} I^{MAB}_f(\rho_{AB}) \quad M_{AB} = \{ P_i^A \otimes P_j^B \} \]

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Pure states: \[ I^A_f = I^{AB}_f = I^B_f = E_f(A, B) \]

All coincident with (generalized) entanglement entropy
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\[ M_{AB} = \{P_i^A \otimes P_j^B\} \]

\[ I_f^{AB}(\rho_{AB}) \geq 0, \text{ with } I_f^{AB}(\rho_{AB}) = 0 \text{ iff } \rho_{AB} = \sum_{i,j} p_{ij} P_i^A \otimes P_j^B \]

classically correlated state

Pure states: \[ I_f^A = I_f^{AB} = I_f^B = E_f(A, B) \]

All coincident with (generalized) entanglement entropy

Mixed states: \[ I_f^{AB} \geq I_f^B, I_f^{AB} \geq I_f^A, I_f^A \neq I_f^B \text{ in general} \]
Special cases:
Special cases:
Special cases:

von Neumann entropy \((S_f = S)\):

\[
I_1^B(\rho_{AB}) = \min_M I_1^{MB}(\rho_{AB}) = \min_{\rho_{AB}^d} S(\rho_{AB} || \rho_{AB}^d)
\]
Special cases:

von Neumann entropy ($S_f = S$):

$$I^B_1(\rho_{AB}) = \min_{M_B} I^M_1(\rho_{AB}) = \min_{\rho^d_{AB}} S(\rho_{AB} || \rho^d_{AB})$$

$\rho^d_{AB}$ arbitrary state of the form $\sum_j q_j \rho_{A/j} \otimes P^B_j$. 
Special cases:

von Neumann entropy ($S_f = S$):

$$I_1^B(\rho_{AB}) = \min_M I_1^{MB}(\rho_{AB}) = \min_{\rho_{AB}^d} S(\rho_{AB}||\rho_{AB}^d)$$

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$I_1^B$, $I_1^{MAB}$ coincide with “quantum discord” of Vedral et al, PRL 104, (2010)
Special cases:

von Neumann entropy ($S_f = S$):

$$I^B_1(\rho_{AB}) = \min_{M_B} I^{MB}_1(\rho_{AB}) = \min_{\rho^d_{AB}} S(\rho_{AB} || \rho^d_{AB})$$

$\rho^d_{AB}$ arbitrary state of the form $\sum_j q_j \rho_{A/j} \otimes P^B_j$.

$I^B_1, I^{MB}_1$ coincide with “quantum discord” of Vedral et al, PRL 104, (2010)

Original Quantum Discord (Ollivier and Zurek 2001):

$$D^B = \min_{M_B} [S(A|B_{MB}) - S(A|B)]$$

$$= \min_{M_B} [I^B_1(\rho_{AB}) - I^B_1(\rho_{B})]$$
Special cases:

von Neumann entropy ($S_f = S$):

$$I_1^B(\rho_{AB}) = \min_{M_B} I_1^{MB}(\rho_{AB}) = \min_{\rho_{AB}^d} S(\rho_{AB}||\rho_{AB}^d)$$

$\rho_{AB}^d$ arbitrary state of the form $\sum_j q_j \rho_{A/j} \otimes P_j^B$.

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Original Quantum Discord (Ollivier and Zurek 2001):

$$D_B = \min_{M_B} [S(A|B_{MB}) - S(A|B)]$$

$$= \min_{M_B} [I_1^B(\rho_{AB}) - I_1^B(\rho_B)]$$

Positivity of $D_B$ based essentially on von Neumann entropy
Special cases:
Special cases:

Linear entropy ($S_f = S_2$):

\[
I_2^B (\rho_{AB}) = \min_{M_B} I_2^{MB} (\rho_{AB}) = \min_{\rho_{dAB}^{\rho_{dAB}}} ||\rho_{AB} - \rho_{AB}^d||^2
\]
Special cases:

Linear entropy \((S_f = S_2)\):

\[
I_2^B(\rho_{AB}) = \min_{M_B} I_2^{MB}(\rho_{AB}) = \min_{\rho^d_{AB}} ||\rho_{AB} - \rho^d_{AB}||^2
\]

Special cases:

Linear entropy ($S_f = S_2$):

$$I^B_2(\rho_{AB}) = \min_{M_B} I^M_2(\rho_{AB}) = \min_{\rho^d_{AB}} \| \rho_{AB} - \rho^d_{AB} \|^2$$


$I^B_2$ suitable for analytic evaluation
Special cases:

Linear entropy ($S_f = S_2$):

$$I_2^B(\rho_{AB}) = \min_{M_B} I_2^{MB}(\rho_{AB}) = \min_{\rho_{dAB}^d} ||\rho_{AB} - \rho_{dAB}^d||^2$$


$I_2^B$ suitable for analytic evaluation

Closed expression for any state of two qubits available
Example: Pure state + max. mixed state
Example: Pure state + max. mixed state

\[ \rho_{AB} = x|\Psi_{AB}\rangle\langle\Psi_{AB}| + \frac{1-x}{d} I_A \otimes I_B \]

\[ |\Psi_{AB}\rangle = \sum_k \sqrt{p_k} |k_A\rangle |k_B\rangle \]
Example: Pure state + max. mixed state

\[ \rho_{AB} = x|\Psi_{AB}\rangle \langle \Psi_{AB}| + \frac{1-x}{d} I_A \otimes I_B \]

\[ |\Psi_{AB}\rangle = \sum_k \sqrt{p_k} |k_A\rangle |k_B\rangle \]

Universal minimum of \( I_f^B, I_f^A \) and \( I_f^{AB} \): Schmidt basis
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**Universal** minimum of \( I_f^B, I_f^A \) and \( I_f^{AB} \): Schmidt basis

\[ \rho'_{AB} = x \sum_k p_k P_k^A \otimes P_k^B + \frac{1-x}{d}I_A \otimes I_B \]

Least mixed state after local measurement!
**Example: Pure state + max. mixed state**

\[ \rho_{AB} = x |\Psi_{AB}\rangle \langle \Psi_{AB}| + \frac{1-x}{d} I_A \otimes I_B \]

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**Universal** minimum of \( I_f^B \), \( I_f^A \) and \( I_f^{AB} \): Schmidt basis

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Leads to analytic evaluation of \( I_f^\nu \), with \( I_f^B = I_f^A = I_f^{AB} \)
Example: Pure state + max. mixed state

\[ \rho_{AB} = x|\Psi_{AB}\rangle\langle \Psi_{AB}| + \frac{1-x}{d} I_A \otimes I_B \]

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Universal minimum of \( I_f^B, I_f^A \) and \( I_f^{AB} \): Schmidt basis

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universal quadratic increase for \( x \to 0 \):
Example: Pure state + max. mixed state

\[ \rho_{AB} = x|\Psi_{AB}\rangle\langle\Psi_{AB}| + \frac{1-x}{d} I_A \otimes I_B \]

\[ |\Psi_{AB}\rangle = \sum_k \sqrt{p_k}|k_A\rangle|k_B\rangle \]

Universal minimum of \( I_f^B, I_f^A \) and \( I_f^{AB} \): Schmidt basis

\[ \rho_{AB}' = x \sum_k p_k P_k^A \otimes P_k^B + \frac{1-x}{d} I_A \otimes I_B \]

Least mixed state after local measurement!

Leads to analytic evaluation of \( I_f^\nu \), with \( I_f^B = I_f^A = I_f^{AB} \)

universal quadratic increase for \( x \rightarrow 0 \):

\[ I_f^\nu(x) \approx \frac{1}{2} x^2 |f''\left(\frac{1}{n}\right)|\left(1 - \sum_k p_k^2\right) \]
Example: Pure state + max. mixed state

\[ \rho_{AB} = x |\Psi_{AB}\rangle \langle \Psi_{AB}| + \frac{1-x}{d} I_A \otimes I_B \]

\[ |\Psi_{AB}\rangle = \sum_k \sqrt{p_k} |k_A\rangle |k_B\rangle \]

Universal minimum of \( I_f^B \), \( I_f^A \) and \( I_f^{AB} \): Schmidt basis

\[ \rho'_{AB} = x \sum_k p_k P_k^A \otimes P_k^B + \frac{1-x}{d} I_A \otimes I_B \]

Least mixed state after local measurement!

Leads to analytic evaluation of \( I^\nu_f \), with \( I_f^B = I_f^A = I_f^{AB} \)

Universal quadratic increase for \( x \rightarrow 0 \):

\[ I_f^\nu(x) \approx \frac{1}{2} x^2 |f''(\frac{1}{n})|(1 - \sum p_k^2) \]

Entanglement needs finite threshold: \( \rho_{AB} \) separable for \( x < \frac{1}{d-1} \)

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Example: Pure state + noise

\[ \rho_{AB} = x|\Psi_{AB}\rangle\langle\Psi_{AB}| + \frac{1-x}{d}I \]

\[ |\Psi_{AB}\rangle = \sqrt{p}|00\rangle + \sqrt{1-p}|11\rangle \]
Example: Pure state + noise

$$\rho_{AB} = x |\Psi_{AB}\rangle \langle \Psi_{AB}| + \frac{1-x}{d} I$$

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**Example: Pure state + noise**

\[
\rho_{AB} = x|\Psi_{AB}\rangle\langle\Psi_{AB}| + \frac{1-x}{d}I
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\[
|\Psi_{AB}\rangle = \sqrt{p}|00\rangle + \sqrt{1-p}|11\rangle
\]

Linear entropy case:

\[
I_2(x) \geq C^2(x) \forall x
\]
Example: Pure state + noise

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Linear entropy case:

\[ I_2(x) \geq C^2(x) \forall x \]

Von Neumann case:

\[ I_1(x) \succeq E(x) \]
Example: Pure state + noise

$$\rho_{AB} = x|\Psi_{AB}\rangle\langle\Psi_{AB}| + \frac{1-x}{d} I$$

$$|\Psi_{AB}\rangle = \sqrt{p}|00\rangle + \sqrt{1-p}|11\rangle$$

Linear entropy case:

$$I_2(x) \geq C^2(x) \forall x$$

Von Neumann case:

$$I_1(x) \geq E(x)$$

$$S_q$$ case:

$$I_q(x) \geq E_q(x)$$ for $$q \in [1.27, 3.5]$$
Example: Decoherence of a Bell state
Example: Decoherence of a Bell state

\[
\rho_{AB} = \frac{1}{2} [\vert 00 \rangle \langle 00 \vert + \vert 11 \rangle \langle 11 \vert + z (\vert 00 \rangle \langle 11 \vert + \vert 11 \rangle \langle 00 \vert )]
\]

\[
= \frac{1+z}{2} \vert \Psi_+ \rangle \langle \Psi_+ \vert + \frac{1-z}{2} \vert \Psi_- \rangle \langle \Psi_- \vert, \quad \vert \Psi_\pm \rangle = \frac{\vert 00 \rangle \pm \vert 11 \rangle}{\sqrt{2}}
\]
Example: Decoherence of a Bell state

\[ \rho_{AB} = \frac{1}{2} [ |00><00| + |11><11| + z(|00><11| + |11><00|)] \]

\[ = \frac{1+z}{2} |\Psi_+\rangle\langle\Psi_+| + \frac{1-z}{2} |\Psi_-\rangle\langle\Psi_-|, \quad |\Psi_\pm\rangle = \frac{|00\rangle \pm |11\rangle}{\sqrt{2}} \]

\[ I_1(z) \leq E(z) \text{ but } I_2(z) = C^2(z) \]
Example: Decoherence of a Bell state

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\rho_{AB} = \frac{1}{2} [ |00><00| + |11><11| + z(|00><11| + |11><00|)]
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\]

\[
I_1(z) \leq E(z) \text{ but } I_2(z) = C^2(z)
\]

\[
I_q(z) \geq E_q(z) \text{ for } 2 < q < 3
\]
Mixture of non-orthogonal states
Mixture of non-orthogonal states

Aligned states of two spins $| \uparrow \downarrow \rangle = | \theta \theta \rangle$, $| \downarrow \uparrow \rangle = | - \theta - \theta \rangle$, with $| \theta \rangle = \exp[-i \theta s_y]| \uparrow \rangle$.
Mixture of non-orthogonal states

Aligned states of two spins $|\uparrow\uparrow\rangle = |\theta\theta\rangle$, $|\downarrow\downarrow\rangle = |\theta - \theta\rangle$, with $|\theta\rangle = \exp[-i\theta s_y]|\uparrow\rangle$

$$
\rho_{AB}(\theta) = \frac{1}{2} (|\theta\theta\rangle\langle\theta\theta| + |\theta - \theta\rangle\langle\theta - \theta|)
$$
Mixture of non-orthogonal states

Aligned states of two spins $|\uparrow\uparrow\rangle = |\theta\theta\rangle$, $|\downarrow\downarrow\rangle = |\theta - \theta\rangle$, with $|\theta\rangle = \exp[-i\theta s_y]|\uparrow\rangle$

$$\rho_{AB}(\theta) = \frac{1}{2}(|\theta\theta\rangle\langle\theta\theta| + |\theta - \theta\rangle\langle\theta - \theta|)$$

Represents pair state in finite $XY$ spin arrays in the vicinity of transverse factorizing field
Mixture of non-orthogonal states

Aligned states of two spins $|\uparrow\uparrow\rangle = |\theta\theta\rangle$, $|\downarrow\downarrow\rangle = |\theta\theta\rangle$, with $|\theta\rangle = \exp[-i\theta s_y]|\uparrow\rangle$

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Represents pair state in finite $XY$ spin arrays in the vicinity of transverse factorizing field

$\rho_{AB}(\theta)$ separable $\Rightarrow E[\rho_{AB}(\theta)] = 0 \forall \theta$. No entanglement
Mixture of non-orthogonal states

Aligned states of two spins $| \uparrow \uparrow \rangle = |\theta \theta \rangle$, $| \downarrow \downarrow \rangle = | - \theta - \theta \rangle$, with $|\theta \rangle = \exp[-i\theta s_y]| \uparrow \rangle$

$$\rho_{AB}(\theta) = \frac{1}{2}(|\theta \theta \rangle \langle \theta \theta | + | - \theta - \theta \rangle \langle -\theta - \theta |)$$

Represents pair state in finite $XY$ spin arrays in the vicinity of transverse factorizing field $\rho_{AB}(\theta)$ separable $\Rightarrow E[\rho_{AB}(\theta)] = 0 \forall \theta$. No entanglement

However, $I_f$ (and $D$) non-zero! $I_f[\rho_{AB}(\theta)] > 0$ for $\theta \in (0, \pi/2)$
Mixture of non-orthogonal states

Aligned states of two spins $|\uparrow\uparrow\rangle = |\theta\theta\rangle$, $|\downarrow\downarrow\rangle = |-\theta - \theta\rangle$,
with $|\theta\rangle = \exp[-i\theta s_y]|\uparrow\rangle$

$$\rho_{AB}(\theta) = \frac{1}{2} (|\theta\theta\rangle\langle\theta\theta| + |-\theta - \theta\rangle\langle-\theta - \theta|)$$

Represents pair state in finite $XY$ spin arrays in the vicinity of transverse factorizing field

$\rho_{AB}(\theta)$ separable $\Rightarrow E[\rho_{AB}(\theta)] = 0 \forall \theta$. No entanglement

However, $I_f$ (and $D$) non-zero! $I_f[\rho_{AB}(\theta)] > 0$ for $\theta \in (0, \pi/2)$

$I_f[\rho_{AB}(\theta)] = 0$ for $\theta = 0$ (product state) and $\theta = \pi/2$ (classically correlated state: $\langle -\theta | \theta \rangle = 0$ for $\theta = \pi/2$)

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Mixture of non-orthogonal states
Mixture of non-orthogonal states
Mixture of non-orthogonal states

Discord as a function of angle $\theta$

$D$ is maximum for $\theta = \theta_m \approx 1, 15\pi/4$
Mixture of non-orthogonal states
Mixture of non-orthogonal states
Mixture of non-orthogonal states

Generalized information loss $I_f^B = I_q^B$ as a function of angle $\theta$
Entanglement and Discord of spin pairs in a spin chain
Entanglement and Discord of spin pairs in a spin chain

Finite cyclic spin $1/2$ chain
with $XY$ couplings in a transverse field
Entanglement and Discord of spin pairs in a spin chain

Finite cyclic spin 1/2 chain
with $XY$ couplings in a transverse field
Entanglement and Discord of spin pairs in a spin chain

Finite cyclic spin $1/2$ chain
with $XY$ couplings in a transverse field

$$H = b \sum_{i} s_{i}^{z} - \frac{1}{2} \sum_{i \neq j} (v_{ij}^{x} s_{i}^{x} s_{j}^{x} + v_{ij}^{y} s_{i}^{y} s_{j}^{y})$$

$$= b \sum_{i} s_{i}^{z} - \frac{1}{2} \sum_{i \neq j} [v_{ij}^{+} s_{i}^{+} s_{j}^{-} + \frac{1}{2} v_{ij}^{-} (s_{i}^{+} s_{j}^{+} + h.c.)]$$
Results
Results
Quantum Discord and entanglement of contiguous pairs vs. transverse field for first neighbor $XY$ couplings

anisotropy $v_y/v_x = 1/2$ and $n = 100$ spins
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Discord and Entanglement of second neighbors in the same chain
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![Graph showing the relationship between D/E and b/vx with labels n=100, L=3. The graph includes data for D and E. The x-axis represents b/vx ranging from 0 to 2, and the y-axis represents D/E ranging from 0 to 0.1.]
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Discord and entanglement of third neighbors in the same chain
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Discord of spin pairs for separations \( L = 1, \ldots, 50 \)
in the same chain
Factorizing field
Factorizing field

At $b = b_s = \sqrt{v_yv_x}$, the chain exhibits a fully separable (and degenerate) exact GS: $|\Psi_0\rangle = |\pm \Theta\rangle$,

$|\Theta\rangle = |\theta\theta \ldots \theta\rangle$, $\cos \theta = \sqrt{v_y/v_x}$

$b_s$ corresponds to last parity ($e^{i\pi S_z}$) GS transition.

Plays the role of a QPT in finite system.

$|\Theta\rangle$ breaks parity symmetry. Definite parity GS’s (RCM 2008):

$|\Theta^\pm\rangle = \frac{|\Theta\rangle \pm |\Theta\rangle}{\sqrt{2(1 \pm \langle\Theta|\Theta\rangle)}}$

Actual GS in the vicinity of $b = b_s$. Leads to

$\rho_{ij} = \text{Tr}_{i\bar{j}} |\Theta^\pm\rangle\langle\Theta^\pm| \approx \rho_{ij}(\theta)$
Conclusions

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- Spin chains: $I_f^\nu$ of pairs exhibits infinite range in the vicinity of factorizing field $b_s$. Confirmation of $b_s$ as QPT for the finite chain
THANK YOU!


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