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What is mass?

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Abstract
Efforts after Newton to reform the concept of mass have not been entirely successful. The unitary Newtonian concept has now fragmented into various 'masses', including inertial mass, active gravitational mass and passive gravitational mass. This paper attempts to clarify the concept of mass.

1. Difficulties today with the concept of mass

According to George Burniston Brown, in the *American Journal of Physics* in 1960 [1],

> nothing in the history of science is perhaps so extraordinary as the doubt and confusion surrounding the concept of mass.

In fact, since the mid-19th century, the Newtonian consensus about mass has been replaced by many definitions. Brown and others criticize as inconsistent the term 'mass' to mean a body, a property of that body and a number [2]. Indeed, it is difficult to imagine more contrasting categories than those of object, property and number. By 1960 mass had fragmented in other directions, into inertial mass, active gravitational mass and passive gravitational mass. Since then, further controversies have arisen [3]. Max Jammer, the leading historian of the concept of mass, wrote in 1999 that [4][1]

> . . . in spite of all the strenuous efforts of physicists and philosophers, the notion of mass, although fundamental to physics, is . . . still shrouded in mystery.

Since mass is the most fundamental concept in mechanics, any obscurity will injure mechanics, and, indeed, much of science. However, the damage done is not as severe as might seem at first sight. An unsatisfactory understanding of mass has little obvious practical impact on physics, since mass is primarily measured in terms of inertia, which is well defined both interpretively and operationally. The technology of the kilogram [5], for example, or the accuracy of the laws containing the concept of mass, do not seem to be affected by debates over the definition of mass.

[1] This reference represents works of extraordinary scholarship.

Also, most experienced physicists seem to be quite comfortable with mass as an everyday working concept, and do not find it ambiguous and fragmented. Perhaps, therefore, it is our definitions that are at fault, rather than the concept of mass itself?

Nevertheless, it troubles physics that mass is poorly understood. For example, it is not possible to tell what avenues of enquiry may be blocked by an obscure concept. Also, the pedagogy of physics suffers from confused concepts.

The creation of a satisfactory definition of mass appears to be a problem of interpretation, rather than one needing new physics to resolve it. However, its long persistence suggests that a new approach is called for. That adopted here attempts, first, to identify the various layers of meaning in today's concept of mass. Then, it reassesses the evidence which justifies this concept.

2. Looking below the surface

Elements of the Newtonian concept of mass survive to this day at a deep level in physics. To clarify mass, therefore, it is necessary to understand the older Newtonian meaning.

Isaac Newton (1642–1727), in his *Philosophiae naturalis principia mathematica* of 1687, states that 'quantity of matter' is synonymous with 'mass' and also with 'body' [6]. Both 'quantity of matter' and 'mass' for Newton meant 'body'—but a body ultimately composed of particles with well-defined volumes, and of the same nature in all objects.

By Newton's day 'quantity of matter' was an established concept, but it was not yet linked to any general procedures of measurement. The vigorous revival of atomism in the 17th century led many natural philosophers to suppose that, at bottom, all bodies are made of particles of the same kind (quantified primary matter), which differ only in volume and shape [7]. With this understanding, 'quantity of matter' was the consolidated volume of its primary particles [8]. There is abundant textual evidence that this was Newton's view [9]:

> . . . the least particles of all bodies [are] all extended, and hard and impenetrable and moveable and endowed with their proper inertia [6].

> . . . there is no difference between . . . bodies but in mere form of matter . . . [10].

> For the matter of all things is one and the same, which is transmuted into countless forms by the operations of nature [11].

> [Those philosophers certainly proceed right who] assume that all matter is homogeneous, and the variety of forms which is seen in bodies arises from some plain and simple relations of the component particles [12].

Roger Cotes (1682–1716), Cambridge mathematician and editor of the second edition of the *Principia*, pointed out to Newton that it is unsafe to assume that 'the Primigenial particles' were 'all of them created equally dense' [13]. As a result, Newton is a little more tentative in the second and third editions about the soundness of his concept of 'quantity of matter'[2]. Nevertheless, he remains confident that it is valid.

Newton uses the term 'mass' in two senses. Sometimes he uses it generically and informally to mean a substantial body or object of unspecified shape[3]. This was the meaning it had outside physics [14]. At other times he uses 'mass' more specifically to mean the body

[2] He writes, for example, '(according to *Aristotle*, *Descartes* and others), . . . there is no difference between . . . bodies but in mere form of matter . . . '[6, p 413]: compare with [10] above.

[3] 'If matter is evenly dispersed throughout an infinite space it could never convene into one mass' [13]; ' . . . the main mass of metal [8, p 532]'.

viewed as an aggregate of hard particles, each of the same nature and density. When using mass in this sense he prefers the term ‘quantity of matter’. Clearly, Newton’s concept of mass made assumptions about the microscopic structure of matter.

For Newton, inertia is an innate property of matter. He states that inertia is directly proportional to the quantity of matter in a body [15]. This was a direct consequence of his belief that all elementary particles are equal in nature and density, and that the inertia of each particle was proportional to its volume.

Newton assumes that the gravity of every body is made up of the gravity of its particles [16]. However, he fully recognizes that he cannot simply assume that bodies of the same weight, but of different natures and textures—he uses the example of wood and gold—will have the same quantities of matter [17]. He adopts a two-stage strategy for discovering how weight and quantity of matter are related. Using pendulums he demonstrates experimentally that all bodies fall with the same acceleration of gravity, independent of their weight, nature or texture. He now argues, applying his second law of motion, that the ratio of the forces of gravity acting on any two bodies, at a given location, will be equal to the compound ratio of their quantities of matter, and their accelerations. But given that the accelerations are equal, he can now conclude that the ratio of weights at a given location is equal to the ratio of quantities of matter [18].

To modern eyes, what Newton proved conclusively here was only that the ratio of weights is equal to the ratio of inertias [19]. Nevertheless, given Newton’s lights, this was a very good argument, and it convinced him, and his successors, of the proportionality of quantity of matter and weight at any given location.

Newton could now choose any body as containing unit quantity of matter, and determine the number of such units in any other body simply by weighing it against his unit. For the first time in the history of science all bodies could now be thought of as made of a common ‘stuff’, which was clearly interpreted, and which could easily be measured. It is important also to note that Newtonian quantity of matter, in its primary sense, is never measured directly; it is always measured indirectly. For any two bodies, the ratio of their quantities of matter is equal to the ratio of their inertias, or to their weights, to their gravitational powers.

In the Newtonian tradition, therefore, the quantity of matter in a body controls the inertia of that body, its weight, and also its gravitational action on other bodies [20]. This made it of fundamental importance in interpreting, unifying and predicting mechanical phenomena.

3. Euler introduces an inertial concept of mass

The terms ‘quantity of matter’ and ‘mass’ remain virtually synonymous from the time of Newton to the late 19th century, and, indeed, to a considerable extent, even to the present. However, ‘quantity of matter’ was given three quite different meanings in 18th century physics: that of Newton [21], a much simpler interpretation as the number of identical particles or atoms in a body [22], and a different kind of interpretation introduced by the great Swiss investigator, Leonhard Euler (1707–1783).

In 1745 Euler quantified inertia more explicitly than Newton. He compares the inertias of two bodies by comparing the forces required to accelerate them [23]. His enthusiasm for inertia made it the most important mechanical property of a body. Ever since Euler, inertia has had a privileged status in the measurement of mass.

Euler goes further, and identifies mass (and quantity of matter) with ‘quantity of inertia’ [24]. What does he mean? ‘Quantity of inertia’ for Euler is a notional division of the body into particles of equal inertia [25]. A count of these allows him to identify quantity of matter, and mass, with ‘quantity of inertia’. But ‘quantity of matter’ no longer means the volume of primary matter, or the number of identical atoms. Why does Euler do this?

Newton’s concept of ‘quantity of matter’ had geometrized mass, distancing the concept somewhat from its mechanical powers. It was also tied to a hypothesis about ultimate matter. Beginning with Euler, physics began a long journey, which is not yet complete, to make mass explicitly dynamical and less hypothetical.

Euler also introduced an unintended ambiguity into the concept of mass, which persists to the present day. Did the ‘quantity of inertia’ mean the body itself, notionally divided into physical units of equal inertia, or did it mean the abstract inertia (the reluctance to accelerate)?

4. Mach attempts a fundamental reinterpretation of mass

Ernst Mach (1838–1916) was a superior experimentalist, but also challenged some of the deepest intuitions of physics—a more difficult task [26]. From 1868, Mach argued that advances in chemistry made the concept of ‘quantity of matter’—as composed of identical atoms—untenable [27]. Eulerian quantity of matter was, of course, immune from this criticism, but Mach does not refer to it. Through the criticisms of Mach and others, by the end of that century, ‘quantity of matter’ had become a controversial concept, and physics was struggling to find a new definition for mass.

Mensurationally, Mach defines mass entirely kinematically—without any need to measure forces. In Mach’s notation,

$$\frac{m}{m'} = -\left(\frac{\varphi'}{\phi}\right)$$

where φ' and φ are the accelerations which the bodies of mass m and m' produce in each other, respectively, when placed opposite to each other. Mach includes mechanical, electrical and gravitationally produced accelerations [28].

Interpretively, Mach defines mass as ‘a special and distinct property determinative of *accelerations*’ [29] (Mach’s italics). Mach is evidently attempting to merge the roles of inertia and gravitational attraction in a single mass concept. He also attempts to remove any association of mass with quantity of matter. For the first time in physics, therefore, mass is explicitly defined abstractly—as a dynamic property of the body—and not as the body itself interpreted as an aggregate of particles. Mach is attempting a colossal shift in interpretation.

The most widely read British writers to take up these ideas, the distinguished mathematicians William Kingdon Clifford (1845–1879) and Karl Pearson (1857–1936), were more radical than Mach. Adopting Mach’s measure of masses, Pearson writes in 1889 [30]:

> . . . mass is a mere number representing a ratio of accelerations. We have here then a particularly clear and intelligible definition.

This type of definition of mass never really caught on in physics, perhaps because it is too reductive and abstract. Also, it seems to define mass as acceleration (a kinematic effect), rather than as the cause of these accelerations.

5. Euler’s mass concept reappears in the 20th century, but fragments

Neither the exclusively kinematic measurements of mass introduced by the 19th century reformers, nor their suggested alternatives to the Newtonian interpretation of mass, were taken up widely in physics. The resulting vacuum of meaning was filled, in the early 20th century, by two new concepts of mass: that which identified mass with ‘inertia’; and the concepts 'inertial mass' and 'gravitational mass', which were given prominence by Albert Einstein (1879–1955).

Mass redefined as 'inertia' was the first to appear in textbooks. How did this come about? We have seen that Euler defined 'mass' as 'quantity of inertia' [25]. This achieved some currency within physics and engineering during the 19th century [31]. When Newtonian quantity of matter became controversial in the late 19th century, many physicists seem to have turned to Euler, and defined mass simply as 'inertia'. However, as we have seen, Euler's term 'quantity of inertia' was ambiguous. In fact, 'inertia', as an abstract property, became the most common meaning of mass. Nevertheless, Euler's original interpretation is found to this day in some definitions of mass. For example [32]:

> The mass of an object is a measure of the amount of matter it comprises, or, in other words, its inertia.

A telling episode in the emergence of mass understood as inertia is found in a tense exchange in 1897 between George Francis Fitzgerald (1851–1901) in Dublin, and his friend Oliver Lodge (1851–1940) in Liverpool. Fitzgerald criticizes 'quantity of matter' as a 'metaphysical notion' and asks [33]:

> Why not call it inertia when it is inertia that is meant, and drop out of use that word mass round which such a tissue of indistinct and obscure ideas have grown . . . ?

Lodge responded sharply to Fitzgerald [34]:

> Why is it that quantity of heat, quantity of electricity are acceptable, but not quantity of matter? Why is it that the quantity called 'matter' cannot legitimately be measured by one of its inalienable properties, inertia?; is conservation of matter a wholly metaphysical and confusing idea?

This debate between Lodge and Fitzgerald illustrates the rising conflict between the old and rhetorically polished definition of mass as quantity of matter, and the new but poorly articulated dynamic concept of mass, which was struggling to replace it.

Einstein gave currency to the terms 'inertial mass' and 'gravitational mass' from 1907, thereby launching a new approach to a dynamic concept of mass [35]. I have been unable to establish who first introduced these terms, but they appear to be quite new [36]. The corresponding concepts, however, were not entirely new. James Wood (1760–1839), Master of St John's College Cambridge, in 1824 defined mass following Euler as 'the aggregate of particles, each of which has a certain degree of inertia'. He also states that mass could be equally well defined as the aggregate of particles, each of which has a certain degree of weight [33]. Did Wood's extension of Euler's concept of mass influence Einstein?

To establish the origins and exact meaning of Einstein's inertial and gravitational mass is a difficult and highly specialized historical and exegetical task, which will not be attempted here. Instead, I will try to outline the meaning of these concepts in today's physics. First, the mensurational definitions of these 'masses' will be examined, since this provides a foundation for physical interpretation.

Take the standard object at Sèvres as a convention, whether measured inertially or gravitationally. Take an unknown mass and the prototype, measured inertially, compressed against a spring in Mach's manner, in carefully controlled conditions. After freeing them, each acquires an acceleration ratio inversely proportional to their inertial masses. Similarly, the same unknown mass and the prototype are compared using, say, a steelyard balance. There are strong experimental grounds for supposing that these two measures are always precisely equal numerically [37]. However, this result cannot be deduced from Newton's mechanics.

Hermann Bondi developed this classification further in 1957, by adding the term 'active gravitational mass' and re-describing Einstein's 'gravitational mass' as 'passive gravitational mass' [38]. The active gravitational mass of the same unknown body can be compared with the prototype by a null method (with appropriate refinements) by comparing gravitational fields. Bondi's distinction is helpful because many textbooks do not discriminate between these very different gravitational methods of specifying mass [39].

These measuring definitions are clear enough, but they still do not fully explain how these three 'masses' are to be interpreted physically. Are they understood as three mechanical properties of the body, that is, as inertia, gravitational attraction and weight, respectively? or do they mean the body itself as an agent of inertia and gravitational attraction, and as the controlling agent of weight, respectively? We can tentatively answer this question by examining how these three mass concepts are actually used in physics.

Take 'inertial mass'. It makes sense to say that 'two inertial masses collide', but it makes no sense to say that 'two inertias collide'. Since only bodies collide, and not abstract properties, this surely means that 'inertial mass' is understood to mean 'body'. We can also say that inertial mass A is twice inertial mass B, so inertial mass is clearly also a quantity. Inertial mass, therefore, seems to mean the body viewed as an inertial agent, and is measured by comparing it with the standard kilogram, viewed as the unit inertial agent. This, of course, is quite close to Euler's concept of 'quantity of inertia'. Analogous considerations apply to the other two 'masses'. If this interpretation is correct, then the masses of the Einstein–Bondi tradition do not mean a triad of properties: each 'mass' means the body itself interpreted as a mechanical agent of a particular kind.

The quantities of physics are, of course, not simply properties—such as a temperature of 273 K. They include actions (work), flowing processes (electric currents), objects described by volume (3 l of water), objects quantified by the number of particles (100 mol of uranium) and objects quantified as agents. The latter is common in physics. A 3 MΩ resistor is an object with a certain electrical resistance. A subatomic charge means a body with a certain electrical attraction. A 20 candela lamp is an object with a certain brightness. These do describe bodies selectively and quantitatively, but not abstractly. Whatever their category, all of these quantities are, of course, represented in exactly the same manner when it comes to calculation—by abstracted measuring numbers.

The proper measure of quantified objects is in terms of other objects chosen as units, such as the use of a resistance box in a bridge to measure an unknown resistance. Also, such measurement involves comparison or null methods, and rarely requires the absolute measure of the property controlled by the object. This explains the measure of the Einstein–Bondi masses by comparison with standard kilograms, where the kilogram is, of course, understood as a physical object.

The Einstein–Bondi definitions of mass appear to be the most authoritative in advanced physics today. Significantly, they have returned to the Newtonian tradition to the extent that mass again means the body itself, described in a special manner, rather than an abstract property. Also, Einstein and Bondi clearly regard the Eulerian inertial definition of mass as too narrow, since they have added definitions in terms of two other mechanical properties, namely, weight and gravitational attraction.

Although there are important similarities, there are also major differences with Newton. Clearly, the Einstein–Bondi masses represent a much more dynamic concept of mass than 'quantity of matter', and they are not tied to any microscopic theory of matter. Also, in the tradition of Einstein, each body has three 'masses', as the agent of inertia, or gravitational attraction or weight control, respectively. By contrast, in the tradition of Newton, these 'masses' are seen as different indirect ways of measuring the same unique quantity, mass, and not as three distinct 'masses'.

The Einstein–Bondi tradition is coherent, it is useful, it is less hypothetical than that of Newton (and yet it fits in well with the Newtonian background), and it offers a richer concept of mass than inertia. Nevertheless, it has serious weaknesses. It undermines the unity of the concept of mass by fragmenting it into three concepts. Indeed, to say that a given body has three 'masses' seems rather extravagant and contrived. Furthermore, using this approach, what is to stop us, as with Bondi, creating further 'masses'? Even in the unlikely event of a proof that the 'masses' are always exactly equal numerically, these three masses would still remain qualitatively different, and would not reduce to a single concept. For those who believe that mass is, or should be, a single concept, this tradition does not yet provide a fundamental definition of mass.

Einstein made another highly important contribution to a more dynamic concept of mass, by stating that mass is 'a reservoir of energy', and by insisting on the 'equivalence' of mass and energy [40]. Energy is not yet incorporated into any well-developed definition of mass, but any acceptable upgrading of this definition must surely do so.

Further insights can be gained into today's interpretation of mass by examining the textbook tradition.

6. Textbook definitions of mass: 1880–2004

Studies of textbook definitions of mass have been carried out by various authors [41]. I have made use of their results, but also conducted my own survey from 1880 to 2004, looking at more than 300 general physics textbooks, at school, technical and university levels, and also at physics dictionaries.

This survey examined the frequency of occurrence, as a definition of mass, of quantity of matter, inertia, the triad of masses deriving from Einstein, mass as a numerical coefficient, and mass defined operationally without explanation. The latter two are rare throughout the period, most authors attempting some physical interpretation of mass. Quite a number of authors, in Euler's manner, combine the first two definitions.

The survey also suggests that 'mass', for some physicists, in certain macroscopic contexts, still means the aggregate of particles or 'structural elements' making up a body [42]. Although not sharply defined quantitatively, this is qualitatively clear and useful, and it can be quantified in various informal and approximate senses. For example, the number of nucleons in a nucleus, or in a macroscopic body, is a rough interpretation for 'quantity of matter'.

What rises quite suddenly to prominence, at the beginning of the twentieth century, is the definition of mass as 'inertia', competing vigorously with 'quantity of matter'. In my sample, only in the 1960s does the inertial definition of mass decisively displace 'quantity of matter' in university level textbooks. Nevertheless, notwithstanding a century and a half of severe criticism, mass defined as 'quantity of matter' stubbornly persists.

To this day, 'quantity of matter' is the most popular definition of mass in elementary physics textbooks, and is used at all levels in chemistry textbooks. It is also sometimes found in university textbooks and physics dictionaries. Despite a general recognition that it is not a precise definition, 'quantity of matter' is often seen as more intuitive than 'inertia' [43], and this must partly explain its continued popularity in pedagogical literature. But it is used outside this context as well.

If a body is uniform, then the old view of mass, as an aggregate of equal particles, applies. Also, mass is then accurately proportional to the quantity of matter. Mass interpreted as volume requires additional controls on density and temperature, but even an approximate measure of mass as volume of matter can be useful. A NASA spokesman stated recently that muscle mass and bone mass are lost during space journeys. The loss of mass here clearly means the loss of a quantity of matter. The 335 billion kilograms of hydrogen burned by the Sun per second gives information about the changing amount of hydrogen in the Sun. For those familiar with handling, say, 500 gm of copper, mass will be associated with a certain volume of copper.

Mass as quantity of matter is well established, therefore, in various macroscopic settings in today's physics, but not as a generally applicable concept of mass. However, even in advanced physics, 'quantity of matter' is sometimes interpreted as if it were a uniform substance or 'stuff' common to all physical particles [44]. On present understanding this seems to be false, but it may also express an intuition that matter, at some deep level, *is* uniform and quantified. Intuition is not proof, and it can be misleading. Nevertheless, echoing Lodge, is it wholly meaningless to say, for example, that a proton contains 1836 times more matter than an electron?

Turning next to mass interpreted as inertia, we find various conflicts. Alpheus Smith (1876–1968) in 1938, and still obviously a comfortable Newtonian, writes in his *Elements of Physics* that '... mass is in a sense the body itself as the amount of matter which it contains' [45]. To re-interpret mass as 'inertia' removes it from its original concrete category of 'body' and places it in the more abstract category of a property. To attempt such a radical change is almost a guarantee of ambiguity. How well, then, has inertia established itself as the working concept of mass in today's physics?

Today, 'mass' continues to be understood generically in physics to mean 'matter' or 'body': a mass *is* a body. For example, when we say that a mass m_1 collides with a mass m_2, we mean a 'body m_1 collides with a body m_2', we do not mean that 'inertia m_1 collides with inertia m_2', which is meaningless. But even when used quantitatively 'mass' usually means 'matter'. 'Density' means the density of matter, not inertia. The equivalence of mass and energy does not mean the equivalence of inertia and energy. The supposed 'unseen mass' in the cosmos does not primarily mean 'unseen inertia'. There is an ancient, deeply rooted and widespread usage of the term 'mass' to mean body or matter, both inside and outside physics, and this is not going to change. The most common understanding of mass in physics, therefore, is now frequently in conflict with the definition of mass as inertia.

There is no difficulty in saying that mass is measured by inertia, or that mass has inertia, the difficulty arises when mass is *reduced* to inertia. It is surely significant that authors who define mass as 'inertia' early in a textbook rarely, if ever, go on to use it consistently to mean a property. They usually revert to the traditional interpretation of mass to mean matter or body. For example: 'Thus the mass of a body is a quantitative measure of the property described in everyday language as *inertia*'. Twenty-four pages later the same authors write: 'It can also be shown that the gravitational force exerted on or by any homogeneous system is the same as though the entire mass of the sphere were concentrated in a point at the centre' [46]. 'Entire mass' here clearly means 'entire matter', and not 'entire inertia', and mass is understood as a source of gravity, not of inertia.

This is no criticism of textbook authors, since physics has not yet cleared up these ambiguities. Furthermore, these various considerations do not prove that the identification of mass with inertia is false, only that it does not fit well into physics, and also that, today, inertia is much too narrow a definition of mass. However, as we shall see below, there is a far more serious objection.

In the 1960s, 'inertial mass' and 'gravitational mass' began to appear more frequently in textbooks, sometimes side-by-side with the definition of mass as 'inertia' [47]. This is ambiguous, since the former means 'body' while the latter is an abstract property.

All of this means that three major and mutually incompatible definitions of mass can be found in physics textbooks today, namely ‘quantity of matter’, ‘inertia’ and the triad of meanings deriving from Einstein.

Fortunately, some physics students and most experienced physicists seem to be able to make an intuitive leap through this tangle of competing definitions. Indeed, I believe that when the concept of mass is actually put to work in physics research there is little ambiguity or fragmentation. Consider the following excerpt from a recent (2000) book on advanced particle astrophysics [48]:

> . . . typically, 10–40% of the total mass is in the form of this gas . . . . A mass of the group [of galaxies] can be derived, in which the baryonic part in the form of gas and galaxies is only 4% (maximum 15%), and the rest is dark matter . . . . On the other hand ROSAT observation of a further small galaxy group . . . suggest that at least 13% of the total mass consists of baryonic matter.

The statement that ‘13% of the total mass consists of baryonic matter’, and various equivalent statements, equate ‘mass’ with matter, with objects such as gas clouds, and treat mass as quantified. Mass is understood here, therefore, as the material body, quantified in some way, and interpreted in some special manner, but not as the property inertia. It also conveys a strong sense that mass is a unitary concept, rather than a triad of concepts. These features suggest that the working concept of mass in today’s physics[4] is not adequately represented by ‘inertia’, or by the multiple definitions deriving from Einstein. Perhaps, therefore, we need to take a fresh look at the evidence justifying the concept of mass.

7. Non-Newtonian inertia

In the 1880s two very different theories arose to challenge Newtonian inertia, those of J J Thomson (1856–1940) and Mach. Based on Maxwell’s equations, J J Thomson in 1881 argued that part of the inertia of a convected charge is not material, and is linked to the electromagnetic ether [49]. Max Abraham (1875–1922) in 1902 went further, and argued that the inertia of all bodies is entirely electromagnetic [50]. Abraham’s theory was soon challenged. Henri Poincaré (1854–1912) showed classically in 1906 that the purely electromagnetic electron would explode [51]. Later, the proton, discovered in 1919 [52], has a much greater inertia than the electron, although electrically the polar opposite of the electron. Later still, in 1932, the neutron made a purely electromagnetic inertia even more difficult to accept [53]. However, J J Thomson’s theory, even in today’s form, poses difficulties.

In modern macroscopic theory, a non-material electromagnetic inertia exists, in a clearly defined form, as the energy and inertia of the radiation field. However, a system of charges without significant radiation has many puzzles relating to the material and electromagnetic interaction energies of the system. For example, is the energy of an inductor located in the magnetic field of the coil? The major difficulty with this is that the magnetic field does not do any work. However, with respect to the present study, the interaction energy and inertia of the system are relatively insignificant compared with the material energy and inertia of the system.

In 1883 Mach argued that no body has intrinsic inertia, and that the appearance of inertia is caused by the combined effects, especially, of all distant bodies [54]. Unlike J J Thomson and Abraham, Mach’s theory was entirely speculative: he had no developed quantitative law, no experiments and no mechanism to replace intrinsic inertia. Einstein coined ‘Mach’s

[4] Hermann Bondi, in a private communication, 20 October 2003, suggests this as the ‘primitive concept of mass’.

principle' in 1912, and endorsed it, but gradually lost interest, and eventually rejected it [55]. Subsequently, many interpretations have been proposed for Mach's principle, but it is difficult to sustain it in the absence of secure physical principles.

8. Approaching the concept of mass

This interrogation of basic mechanical concepts will begin with the more directly accessible quantities. For example, in discussing the effect of a contact force on a free body, inertia i immediately controls the ratio of force to acceleration, and the term 'mass' will not be introduced. The prototype kilogram will be taken as the body of unit inertia.

The strength of a body as a gravitational source can be measured by the number of kilograms Γ which act as an equivalent source. In this context the term 'mass' will not be introduced. Sometimes Γ is termed the gravitational 'charge', but this term has strong electromagnetic associations, which are not appropriate here.

Only when fundamental mechanical quantities introduced in this manner have been exhausted, will the term 'mass' be introduced—always respecting the history and associations of this term.

To avoid the possibility of uncertain interpretations, Newton's law of gravity for weak fields will be expressed in its simplest form as follows:

$$G\frac{\Gamma_1}{r^2} = g_1.$$

Here Γ_1, the gravitational source, is quasi-static and small compared with r, while g_1 is the gravitational acceleration that would be experienced by all small, slowly moving bodies, at a distance r from the source.

The assertion that all bodies, irrespective of composition, will experience the same local gravitational acceleration has been tested with increasing accuracy since Galileo (1564–1642). The most recent observations claim an accuracy of 1 part in 10^{13}. Equivalently, this result is termed the 'principle of equivalence', which has an even higher warrant as an axiom of general relativity [56].

If g_1 is the free acceleration experienced by a small, quasi-static gravitating source Γ_2, then the latter will induce an acceleration g_2 in the original source, if it is also free, and this obeys the equation

$$g_2 = G\frac{\Gamma_2}{r^2}.$$

Also,

$$\frac{g_2}{g_1} = \frac{\Gamma_2}{\Gamma_1}.$$

The ratio of the gravitational source strengths of two quasi-static bodies is equal, therefore, to the ratios of their mutual accelerations, a very well-known result[5] [57]. If we now arrest each body and hold them at a fixed distance r apart—by a rod, if the scale is appropriate—then the magnitude of the contact force, required to stop the accelerations, exerted by the rod on each body, is expressed by

$$F_1 = i_1 g_2 = \frac{i_1\Gamma_2}{r^2},$$

[5] This result would follow, even if there were a deviation from Newton's law of gravity, provided the functional relationship is the same for each body [57].

and

$$F_2 = i_2 g_1 = \frac{i_2 \Gamma_1}{r^2}$$

where i_1, i_2 are the linear inertias of the two bodies. These forces must be equal and opposite, otherwise the system would self-accelerate, violating various conservation rules. Therefore,

$$\frac{i_1}{i_2} = \frac{\Gamma_1}{\Gamma_2}.$$

This means that the inertia of every quasi-static body is proportional to its gravitational source strength, also a very well-known result.

The word 'weight' in physics has various meanings. It is helpful here to distinguish between the attraction of gravity acting *on* the body, the primary meaning of weight in physics and the 'dead weight' of an inert body [58], meaning the contact force exerted *by* that body on the supporting body (the weight of Poppy on my lap): this seems to be the everyday meaning of 'weight'. The latter force is, of course, ultimately caused by the gravitational field, but modified by buoyancy and the Earth's rotation [59].

If two arbitrary bodies are in equilibrium on a balance of unequal arms, in idealized circumstances[6], then the support forces exerted by the scale pans on each body, to prevent them accelerating, are $F_1 = i_1 g$ and $F_2 = i_2 g$, respectively, where g is the local acceleration of gravity. But these are equal and opposite to the contact forces W_1 and W_2 immediately exerted by the bodies on the scale pans. Clearly, therefore,

$$W_2 = i_2 g \qquad \text{and} \qquad W_1 = i_1 g$$

and

$$\frac{W_1}{W_2} = \frac{i_1}{i_2}.$$

This means that the ratio of the dead weights of any two static bodies at a given location is equal to the ratio of their linear inertias, also an old result.

Clearly, the experiments and laws considered here lead only to two intrinsic dynamic quantities in a body, namely, linear inertia and gravitational source strength.

9. Body energy

The 19th century reformers of the concept of mass were unaware of a third intrinsic dynamical property of a body, quite distinct from inertia and gravitational attractive power, namely the total energy of the body. In the standard or Thomson–Rankine definition, energy is the capacity of an object or system to perform work [60]. Energy, therefore, is a property of the body or system. The total energy of a body means the total work the body can do in principle, were it fully to exhaust its substance in performing work. This is usually termed its 'mass energy'. 'Mass energy' in this context, therefore, means 'body energy', and the term 'mass' is being used in its generic sense to mean 'body'.

'Body energy' is a particularly intimate property of a body, in that the loss of energy here causes a loss of the very substance of the body. But to go further and say that matter *is* energy is incoherent, because it identifies an object with an abstract property of that object.

Einstein's famous equation $E = mc^2$ implies that, for any two static bodies,

$$\frac{E_1}{E_2} = \frac{m_1}{m_2} = \frac{i_1}{i_2},$$

[6] I am ignoring buoyancy, and the centripetal acceleration of each body towards the centre of its circle of latitude.

since mass is measured by inertia. Physics places enormous confidence in this result, a result which is continuously tested in high energy laboratories around the world. It follows from this and earlier results that, for any two neighbouring static bodies,

$$\frac{E_1}{E_2} = \frac{i_1}{i_2} = \frac{\Gamma_1}{\Gamma_2} = \frac{W_1}{W_2}.$$

This means that the relative values of body energy, inertia, gravitational source strength—and dead weight (at a given location)—are the same in all stationary bodies. Clearly, matter is so constituted that the three intrinsic mechanical quantities (E, i, Γ) have the same relative values in all bodies at rest.

Is the symbol m different from each of these quantities, or identical with one or more of these? Before this critical issue is explored, inertia will be examined more carefully.

10. What is linear inertia?

Inertia is measured conventionally by the ratio of force to acceleration. However, it is well known that if the velocity of a body prior to acceleration is along the z-axis, then the x, y, z components of inertia are γi_0, γi_0 and $\gamma^3 i_0$, respectively, where $\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$, i_0 is the static inertia, $i_l = \gamma^3 i_0$ is the longitudinal inertia and $i_t = \gamma i_0$ the transverse inertia [61].

If the present velocity has an arbitrary direction relative to the reference axes, then $\boldsymbol{F} = \mathbf{i} \cdot \boldsymbol{a} = i_0\gamma^3[(1-\beta^2)\mathbf{u} + \boldsymbol{\beta}\boldsymbol{\beta}] \cdot \boldsymbol{a}$, $\boldsymbol{F}$ is the force, $\boldsymbol{a}$ the acceleration, $\boldsymbol{\beta} = \boldsymbol{v}/c$, and $\mathbf{u} = \boldsymbol{ii} + \boldsymbol{jj} + \boldsymbol{kk}$, is the unit dyadic tensor. $\mathbf{i}$ is the linear inertia tensor, a physical quantity with more degrees of freedom than a vector [62]. It transforms under a rotation of coordinate axes as a second rank tensor.

When $\boldsymbol{v} = 0$ the linear inertia tensor becomes isotropic, and $\mathbf{i} = \mathbf{i}_0 = i_0\mathbf{u}$, where i_0 is the scalar magnitude of this tensor. Other isotropic tensors, such as pressure in a static fluid, or the rotational inertia of a uniform sphere about a central axis, can also be characterized by a scalar measure.

Strictly speaking, therefore, linear inertia $\boldsymbol{i}$, even static linear inertia, is not a true scalar. Consider the relationships $E = \gamma mc^2$ and $\boldsymbol{p} = \gamma m\boldsymbol{v}$, which relate scalars (zero rank tensors), and polar vectors (first rank tensors), respectively. As is well known, both sides of an equation of physics must always be equal numerically, dimensionally, in gauge[7] and in rank. If m is interpreted as the intrinsic linear inertia and is replaced by $\mathbf{i}_0$, both equations fail the fourth test. If γm is replaced by $\mathbf{i}$, both fail the first and the fourth test. While scalar notation can sometimes be used to represent tensors, it must always be possible to rewrite it as tensor notation. This is not possible here. Rest mass and resting inertia are, of course, numerically equal in SI, but mass is a zero rank tensor and inertia a second rank tensor, so they cannot be physically identical.

Although it makes sense to speak of transverse and longitudinal inertia, it does not make sense to speak of transverse and longitudinal mass. If a useful meaning can be given to mass that is consistent with the formalism of mechanics, inertia cannot be that meaning.

General relativity implies that, for a freely moving body in weak fields, the ratio of the momentum developed per second to the local gravitational intensity is expressed by the tensor $\mathbf{w} = \gamma m[(1+\beta^2)\mathbf{u} - \boldsymbol{\beta}\boldsymbol{\beta}]$.[8] Here, m can be read as the scalar value w_0 of this tensor when

[7] Gauge invariance here means that both sides of an equation must remain equal, when the size or gauge of the base units of the system of measurement used is arbitrarily changed.

[8] Based on Okun [3, p 34]. In a gravitational field the momentum developed per second becomes $\frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t} = -G\gamma Mm\frac{\boldsymbol{r}(1+\beta^2)-\boldsymbol{\beta}(\boldsymbol{\beta}\cdot\boldsymbol{r})}{r^3} = -[\gamma m[(1+\beta^2)\mathbf{u} - \boldsymbol{\beta}\boldsymbol{\beta}]] \cdot [GM\frac{\boldsymbol{r}}{r^3}]$.

$\beta = 0$. Like inertia, $\boldsymbol{w}$ becomes isotropic for bodies at rest. However, it is mathematically different from inertia. Also, increasing w_0 does not reduce gravitational acceleration; indeed, it has no effect whatever upon it. It does not seem appropriate, therefore, to call this quantity 'gravitational inertia'. It might be described as 'weight control', or 'capacity for weight'. It is easy to show, for any two bodies at rest, that $\frac{w_1}{w_2} = \frac{W_1}{W_2}$. From this perspective, Bondi's 'passive gravitational mass' means the body as the agent of the intrinsic property w_0.

11. Unifying the concept of mass

For more than two centuries the concept of mass in physics has been on a trajectory towards an increasingly dynamic interpretation. 'Quantity of matter' was replaced by 'inertia', and then by the Einstein–Bondi masses. Energy represents a parallel development of a dynamic concept of mass. Again, important aspects of Newton's concept have survived, especially mass as the quantified material object. Can we find a unified definition of mass, which might bring us closer to the goal of this long development?

Subatomic particles can exist without charge or spin, and may be baryons, mesons or leptons. But all material particles, whatsoever, must have energy, inertia, a gravitational field and a capacity for weight. These are primitive properties common to all objects, and have the same relative values in all bodies at rest. Furthermore, often in nucleosynthesis, for example, energy, inertia and the others reduce in lock step, without any change occurring in the charge or spin, or in the nature of the interacting particles. Clearly, these four intrinsic mechanical properties exist as a tightly coordinated group in all bodies at rest.

Energy appears to be the most fundamental of these properties. Each of the other properties can act without a reduction in strength. However, if a body does work from rest, by any mechanism whatever, it loses substance, the reservoir of energy is depleted and the rest of the core mechanical properties are proportionally reduced. This implies that energy is the fundamental quantitative structure of matter, and controls the values of the other basic mechanical quantities. There are also, of course, equally fundamental qualitative structures that are common to all matter—such as that which makes it matter rather than radiation, for example[9].

Matter bearing the structures common to all material bodies will be termed 'common matter'. This is primarily quantified by energy, that is, as an agent of work. Matter at a fundamental level is quantified, therefore, not by volume or by the number of particles, but dynamically. It has the additional mechanical properties inertia, gravitational attraction and a capacity for weight. These are proportional to body energy, and are controlled by it. 'Common matter' is not a form of matter accessible in a 'bare' state. All observable matter has higher levels of intrinsic structure, which determine particle category, charge and spin, for example.

Here, therefore, we seem to have an appropriate interpretation of mass. Mass means 'common matter', as just described, or the body as composed of common matter. Mass is quantified by energy, but body energies are not easily compared. Physics is entirely confident in the linear proportionality of resting inertia and body energy, so inertia provides an exact indirect method of measuring mass. Physics is confident, to at least 1 part in 10^{13}, of the linear proportionality of inertia with local weight, and also with gravitational attractive power. These also provide fully satisfactory indirect mass measures [5].

This concept and measure of mass is basically a unification of the Einstein–Bondi 'masses' by means of the fundamental concept of body energy. Since there is only one meaning of mass in this interpretation—common matter as quantified by energy—it is not appropriate to

[9] Volume, of course, becomes well defined only at near-macroscopic assemblies of subatomic particles.

speak of three 'masses'. But we can, of course, speak of mass as a reservoir of energy, or as acting inertially, or as a source of gravity, or as having weight.

If this interpretation of the leading concept of mass in today's physics is correct, mass *m*, as the primitive mechanical object, is categorically distinct from E, i and w, which are some of its properties. This justifies the introduction of a distinct concept of mass. It follows, for any two neighbouring bodies at rest,

$$\frac{m_1}{m_2} = \frac{E_1}{E_2} = \frac{i_1}{i_2} = \frac{\Gamma_1}{\Gamma_2} = \frac{w_1}{w_2} = \frac{W_1}{W_2}.$$

The kilogram at Sèvres is the standard unit of mass. It has also been chosen by physics as the body, at rest, which possesses unit inertia, and unit gravitational source strength, but not as the body possessing unit energy[10], or unit weight. This, of course, means that mass, the gravitational source and inertia, can be conveniently represented by the same symbol *m*, and the same numerical values, for a body at rest. Inertia, however, is a tensor, and at relativistic speeds mass *m* and inertia $\boldsymbol{i}$ differ both numerically and structurally, and need to be carefully distinguished.

Physics, of course, already thinks of mass as a source of energy, and as an agent of inertia, gravitational attraction and weight control. Rarely are all of the powers of mass emphasized at once. What is stressed will depend on context. When a NASA spokesman stated recently that mass is expensive to send into space, surely he was thinking of the body as subject to weight and inertia. In fission and fusion research, mass is mainly thought of as a reservoir of energy. In celestial mechanics mass is an agent of gravitational attraction.

To conclude, I will examine an issue that has troubled physics for a century.

12. Is there a 'relativistic' mass? [63]

The significance of *m* for a moving body can be better understood by inspecting the following equations, which express its chief mechanical properties.

$$E = \gamma mc^2$$

$$\boldsymbol{g} = -G\gamma m\frac{\boldsymbol{r}(1+\beta^2) - \boldsymbol{\beta}(\boldsymbol{\beta}\cdot\boldsymbol{r})}{r^3}$$

$$\mathbf{I} = \gamma^3 m[(1-\beta^2)\mathbf{u} + \boldsymbol{\beta}\boldsymbol{\beta}]$$

$$\mathbf{w} = \gamma m[(1+\beta^2)\mathbf{u} - \boldsymbol{\beta}\boldsymbol{\beta}]$$

$$\boldsymbol{p} = \gamma m\boldsymbol{v}.$$

Each equation now relates the intrinsic 'common matter' *m* in the body to one of its mechanical properties. Because of new kinematic factors, the structure of these expressions is more complex than for a body at rest. Nevertheless, each quantity remains strictly proportional to *m*.

Clearly, in bodies moving inertially, the core mechanical properties no longer have fixed ratios to each other and, indeed, become incommensurable, since they change considerably in structure. Also, the only commensurable comparison mass measure possible requires the measuring kilograms to move with the velocity of the test body. We then find, of course, that the number of commoving kilograms *m* with the same basic mechanical properties as the

[10] If mass is quantified by energy, why not choose the unit of mass as that object with unit body energy? This is impractical, since such a body would be 1.11×10^{-17} kg.

test body is exactly the same as when both bodies are at rest. This is because the intrinsic properties of an object do not vary with inertial motion. It also means that mass defined in this manner is a true scalar quantity, and is motionally invariant.

In particular, if we wish to measure relativistic mass in a manner analogous to the measures of relativistic length and time, then it is necessary to measure the mass of the moving body using standard kilograms at rest in the framework we conventionally call at rest. But this is impossible with mass. Clearly, relativistic mass, $m = \frac{m_0}{\sqrt{1-v^2/c^2}}$, or $m = \gamma m_0$, is not a well-defined concept, and seems to be no more than a mathematical artefact[11].

Early in his career Einstein did use 'relativistic' mass. However, in 1948 he wrote that [64]

> ... no clear definition can be given [for the] mass $M = m\left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}$ of a moving body. It is better to introduce no other mass than the 'rest mass' m.

Today, nuclear and particle physics make no reference to relativistic mass. It has also been estimated that about 60% of authors now writing on special relativity do not introduce it [65]. There is a growing tendency to use the symbol m rather than m_0 for rest mass (as I do here), not even to speak of 'rest mass', and to use only one term 'mass'. Nevertheless, some experienced authors are unhappy with any attempt to outlaw relativistic mass [66], and I will now examine some arguments defending it.

The 'cyclotron' angular velocity ω of a particle of charge q circling, in a magnetic field of intensity B seems to be related to the 'relativistic mass' γm, as follows: $\omega = q\frac{B}{\gamma m}$. However, this example illustrates the importance of distinguishing between mass (a scalar), and inertia (a tensor). In the present case, the velocity is perpendicular both to the force and to the acceleration, therefore

$$F = Bqv = i_t a = i_t v\omega,$$

where $i_t = \gamma i_0$ is the transverse inertia of the moving particle, and i_0 of course, is numerically equal to m. It follows that $\omega = q\frac{B}{i_t}$, and relates ω, q, B, i_t, and not explicitly the mass.

When a stationary body is heated its mass increases, and yet this increase is clearly relativistic—a consequence of the increased motion of molecules. If we allow this form of relativistic mass increase, why not allow that γm represents the mass of a translating body? However, as has often been pointed out, a hot body at rest remains inertially and gravitationally isotropic. This means that its mass can be measured unambiguously using standard kilograms at rest, which we cannot do for a translating body. The same considerations apply, of course, to the mass associated with all other internal motions of a body. Indeed, even the prototype kilogram itself has a tiny mass component of this nature.

To sum up, I have argued that textbook definitions of mass do not accurately reflect the working concept of mass in today's physics. I have interpreted this as follows. Generically, mass is used in physics simply to mean 'matter' or 'material object'. Specifically, mass means the 'common matter' in a body, which is quantified by its energy, and which bears the additional mechanical properties inertia, gravitational attraction and a capacity for weight. These are controlled by body energy, are proportional to it and can be used to measure mass. Physics moves easily between the generic and this specific understanding of mass.

It could also be said that mass is the 'quantity of common matter' in a body, with matter quantified by its energy and not by its volume. Although succinct, this says far too little about the other dynamic properties of 'common matter'.

[11] Relativistic gravitational source strength Γ is not well defined, either.

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